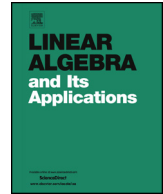




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# Linear Algebra and its Applications

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## On maps preserving operators of local spectral radius zero



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### ARTICLE INFO

#### Article history:

Received 3 July 2016

Accepted 1 October 2016

Available online 4 October 2016

Submitted by P. Semrl

#### MSC:

primary 47B49

secondary 47B48, 47A10, 47A11

#### Keywords:

Linear preserver

Quasi-nilpotent part

Local spectral radius

### ABSTRACT

Let  $\mathcal{L}(X)$  be the algebra of all bounded linear operators on a complex Banach space  $X$ . We describe surjective linear maps  $\phi$  on  $\mathcal{L}(X)$  that satisfy

$$r_{\phi(T)}(x) = 0 \implies r_T(x) = 0$$

for every  $x \in X$  and  $T \in \mathcal{L}(X)$ . We also describe surjective linear maps  $\phi$  on  $\mathcal{L}(X)$  that satisfy

$$r_T(x) = 0 \implies r_{\phi(T)}(x) = 0$$

for every  $x \in X$  and  $T \in \mathcal{L}(X)$ . Furthermore, we characterize maps  $\phi$  (not necessarily linear nor surjective) on  $\mathcal{L}(X)$  which satisfy

$$r_{\phi(T)-\phi(S)}(x) = 0 \text{ if and only if } r_{T-S}(x) = 0$$

for every  $x \in X$  and  $T, S \in \mathcal{L}(X)$ .

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## 1. Introduction

Let  $\mathcal{L}(X)$  be the algebra of all bounded operators on a complex Banach space  $X$ . The local spectral radius of an operator  $T \in \mathcal{L}(X)$  at a point  $x \in X$  is defined by

$$r_T(x) = \limsup_{n \rightarrow +\infty} \|T^n x\|^{\frac{1}{n}}.$$

Recall that the quasi-nilpotent part of an operator  $T \in \mathcal{L}(X)$  is given by

$$H_0(T) := \{x \in X : \limsup_{n \rightarrow +\infty} \|T^n x\|^{\frac{1}{n}} = 0\}.$$

The problem of describing linear or additive maps on  $\mathcal{L}(X)$  preserving the local spectra has been initiated by A. Bourhim and T. Ransford in [5], and continued by several authors; see for instance [2–4,6–8] and the references therein.

In [8], C. Costara described surjective linear maps on  $\mathcal{L}(X)$  which preserve operators of local spectral radius zero at points of  $X$ . He showed that if  $\phi : \mathcal{L}(X) \rightarrow \mathcal{L}(X)$  is a linear and surjective map such that for every  $x \in X$  and  $T \in \mathcal{L}(X)$ , we have

$$r_{\phi(T)}(x) = 0 \text{ if and only if } r_T(x) = 0,$$

then there exists a nonzero scalar  $\mu \in \mathbb{C}$  such that  $\phi(T) = \mu T$  for all  $T \in \mathcal{L}(X)$ .

This result has been extended by Bourhim and Mashregi in [4] where it is shown that if  $\phi$  is a surjective (not necessarily linear) map on  $\mathcal{L}(X)$  that satisfies

$$r_{\phi(T)-\phi(S)}(x) = 0 \text{ if and only if } r_{T-S}(x) = 0,$$

for every  $x \in X$  and  $T, S \in \mathcal{L}(X)$ , then there are a nonzero scalar  $\mu \in \mathbb{C}$  and an operator  $A \in \mathcal{L}(X)$  such that  $\phi(T) = \mu T + A$  for all  $T \in \mathcal{L}(X)$ .

In this paper, we start by studying surjective linear maps  $\phi$  on  $\mathcal{L}(X)$  such that either

$$H_0(\phi(T)) \subset H_0(T)$$

for all  $T \in \mathcal{L}(X)$ , or

$$H_0(T) \subset H_0(\phi(T))$$

for all  $T \in \mathcal{L}(X)$ . This will give characterizations of surjective linear maps  $\phi$  on  $\mathcal{L}(X)$ , that preserve operators of local spectral radius zero in one direction; i.e.

$$r_{\phi(T)}(x) = 0 \implies r_T(x) = 0$$

for every  $x \in X$  and  $T \in \mathcal{L}(X)$ , or

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