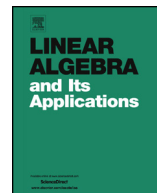




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Contractive maps on operator ideals and norm inequalities II



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ABSTRACT

Let $(\mathcal{I}, ||| \cdot |||)$ be a norm ideal of operators equipped with a unitarily invariant norm $||| \cdot |||$. We exploit integral representations of certain functions to prove that certain ratios of linear operators acting on operators in \mathcal{I} are contractive. This leads to some new and old norm inequalities. We also lift a variety of inequalities to the operator setting, which were proved in the matrix setting earlier.

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1. Introduction

Let $\mathbb{B}(\mathcal{H})$ be the algebra of all bounded linear operators on a complex separable Hilbert space $(\mathcal{H}, \langle \cdot, \cdot \rangle)$. The cone of positive operators is denoted by $\mathbb{B}(\mathcal{H})_+$. We shall

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consider a norm ideal $(\mathcal{I}, |||\cdot|||)$ of $\mathbb{B}(\mathcal{H})$ equipped with a unitarily invariant norm and for notational convenience we shall denote throughout this by \mathcal{I} instead of $(\mathcal{I}, |||\cdot|||)$. As usual, we shall denote the operator norm by $\|\cdot\|$ and by L_X, R_Y the left, right multiplication maps on $\mathbb{B}(\mathcal{H})$ respectively, i.e. $L_X(T) = XT$ and $R_Y(T) = TY$. Since L_X and R_Y commute, we have

$$e^{L_X+R_Y}(T) = e^X T e^Y.$$

More concisely, if $f(x)$ and $g(x)$ are two infinitely many times differentiable, then we have

$$f(L_X)g(R_Y)(T) = f(X)Tg(Y).$$

Let X be selfadjoint member of $\mathbb{B}(\mathcal{H})$ and let $A \in \mathbb{B}(\mathcal{H})$ be arbitrary. Then unitary invariance of the operator norm leads to the elementary inequality

$$|||A \pm iXA||| \geq |||A|||.$$

The above inequality may be written as

$$||(I \pm iL_X)A|| \geq ||A||. \quad (1.1)$$

In 1995, Bhatia and Davis [6] proved a difference version of the Heinz inequality

$$|||A^\alpha XB^{1-\alpha} - A^{1-\alpha}XB^\alpha||| \leq |2\alpha - 1| |||AX - XB|||, \quad (0 \leq \alpha \leq 1) \quad (1.2)$$

for $A, B \in \mathbb{B}(\mathcal{H})_+$ and $X \in \mathcal{I}$.

Later in 1998, Kosaki [18] proved several related inequalities using Poisson integral and Fourier transform. Among those a few are matrix version of the well-known inequalities $|\sin x| \leq |x|$, $|x| \leq |\sinh x|$ and geometric–logarithmic mean inequalities.

In 2005, Jocić [13] proved Cauchy–Schwarz norm inequalities and several other related non-trivial inequalities using Gel’fand’s integration techniques.

Then, in 2014, Kapil and Singh [15] proved that the function,

$$f(\alpha) = |||A^\alpha XB^{1-\alpha} - A^{1-\alpha}XB^\alpha|||$$

is monotonically decreasing for $\alpha \in [0, 1/2]$ and is monotonically increasing for $\alpha \in [1/2, 1]$, see [15, p. 491]. More precisely, it is proved that $f(\alpha)$ is a convex function on $[0, 1]$. For more such inequalities the reader may see [3, 5, 12, 16–18, 20, 23, 24].

To prove these and related inequalities, contractive maps on \mathcal{I} play a key role. This viewpoint has been taken by some other authors. For instance, the authors in [8, 9, 11, 18] proved positive definiteness of the functions such as $\frac{\cosh(\alpha x)}{\cosh x}$, $\frac{\sinh(\alpha x)}{\sinh x}$, for $0 < \alpha < 1$,

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