Linear Algebra and its Applications 513 (2017) 210–223

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Characterizations of matrices with nonzero signed row compound ☆

Ling-Zhi Ren [a,*], Hai-Ying Shan [b]

[a] *Department of Mathematics and Information Science, Yantai University, Yantai 264005, China*
[b] *Department of Mathematics, Tongji University, Shanghai 200092, China*

ARTICLE INFO

Article history:
Received 1 March 2016
Accepted 6 October 2016
Available online 12 October 2016
Submitted by R. Brualdi

MSC:
15A09
15A48

Keywords:
Matrix
Signed row compound
Null-space
Bipartite graph

ABSTRACT

A real square matrix A is called a sign-nonsingular (SNS) matrix if every matrix with the same sign pattern as A is not singular. An $m \times n$ matrix A with term rank m is called to have a nonzero signed row compound provided that each square submatrix of order m of A is an SNS-matrix or has an identically zero determinant. As generalizations of SNS-matrices, S^*-matrices, and totally L-matrices, matrices with nonzero signed row compound have a close relationship with matrices with signed null-spaces which are applied to characterize linear systems with signed solutions. In this paper, matrices with nonzero signed row compound are characterized in terms of their signed bipartite graphs. Following these results, characterizations of matrices with signed null-spaces and full row term ranks in terms of their signed bipartite graphs are obtained too. The recursive structure of $m \times (m+2)$ row sign balanced (RSB) maximal $(0,1,-1)$-matrices with nonzero signed row compound (or with signed null-spaces) is also characterized.

☆ Partially supported by NSFC project No. 11271288.
* Corresponding author.
E-mail addresses: lingzhi123me@163.com (L.-Z. Ren), shan_haiying@tongji.edu.cn (H.-Y. Shan).

http://dx.doi.org/10.1016/j.laa.2016.10.006

1. Introduction

Let $A = (a_{ij})_{m\times n}$ be a real matrix. The zero pattern of A is a $(0, 1)$-matrix $(z_{ij})_{m\times n}$ satisfying $z_{ij} = 1$ if and only if $a_{ij} \neq 0$. The *sign* of a real number a, denoted by $sgn(a)$, is defined as follows: $sgn(a) = 1$, if $a > 0$; $sgn(a) = -1$, if $a < 0$; $sgn(a) = 0$, if $a = 0$. The *sign pattern* of A, denoted by $sgn(A)$, satisfies $sgn(A) = (sgn(a_{ij}))_{m\times n}$. The set of all the matrices with the same sign pattern as A is called the qualitative class of A, denoted by $Q(A)$, i.e. $Q(A) = \{\widetilde{A}|sgn(\widetilde{A}) = sgn(A)\}$. Qualitative matrix theory involves the study of "qualitative properties" of matrices which depend only on the sign patterns of the matrices. Qualitative matrix theory has been extensively studied, see, for example, [2–7].

The study of sign solvable linear systems originated with the work of qualitative economics by Samuelson in 1947 (see [2]). Let A be an $m \times n$ matrix and b an $m \times 1$ vector. The linear system $Ax = b$ is *sign solvable* provided each linear system $\widetilde{A}x = \widetilde{b}$ (where $\widetilde{A} \in Q(A)$ and $\widetilde{b} \in Q(b)$) has a solution and all such solutions have the same sign pattern.

In 2000, Kim and Shader generalized the concept of sign solvable linear system to that of linear system with signed solutions (see [3]). The linear system $Ax = b$ is said to have *signed solutions* provided for each $\widetilde{A} \in Q(A)$ and $\widetilde{b} \in Q(b)$, the set of sign patterns of the solutions of $\widetilde{A}x = \widetilde{b}$ is equal to that of $Ax = b$.

A matrix A is defined to have *signed null-space* provided $Ax = 0$ has signed solutions. In [3], matrices with signed null-spaces were introduced and applied to characterize linear systems with signed solutions.

A real matrix is an *L-matrix* provided every matrix in its qualitative class has linearly independent columns (note that it is defined in terms of rows in [2]). A real square matrix A is called a sign-nonsingular (SNS) matrix if every matrix in $Q(A)$ is not singular. An m by $m + 1$ matrix A is called an S^**-matrix* provided that each submatrix of order m of A is an SNS-matrix. It is known that there exists a row of an S^*-matrix with exactly two nonzero entries (see [2, Theorem 4.1.1]). An m by n matrix A is called a *totally L-matrix* provided that each submatrix of order m of A is an SNS-matrix. L-matrices, SNS-matrices, S^*-matrices and totally L-matrices are matrices with signed null-spaces (see [3]).

An $m \times n$ matrix A with term rank m is called to have a *nonzero signed row compound* [6] (also called nonzero signed mth compound in [2]) provided that each square submatrix of order m of A is an SNS-matrix or has an identically zero determinant. Matrices with nonzero signed row compound are generalizations of SNS-matrices, S^*-matrices and totally L-matrices.

A matrix A is said to be *permutation equivalent* to matrix B, if A can be transformed into B by permuting its rows and columns.

In 1995, Brualdi and Shader pointed out in [2, Section 5.3] that a matrix with nonzero signed row compound is permutation equivalent to a matrix with a special block par-