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Positivity and stability analysis for linear implicit difference delay equations

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ABSTRACT

This paper deals with positivity and stability of linear implicit difference delay equations. Being different from the Lyapunov function approach commonly used in stability analysis, the method employed in this paper gives a way to solve the exponential stability of linear implicit difference equations with time-varying delay. By decomposition state-space and mathematical induction method, new necessary and sufficient conditions for positivity and stability of such systems are derived. Numerical examples are given to illustrate the proposed results.

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1. Introduction

During the past few decades, a considerable amount of research has been done in the field of stability of singular systems with delays [1–6]. Compared with the regular systems, the stability problem of singular systems, in particular, of positive singular systems, is much more complicated since regularity and absence of impulses are necessary to be considered simultaneously. Positive systems, whose state variables naturally take nonnegative values, have received considerable attention because of their engineering insight in communication systems, formation flying, and systems theory [7,8]. In positive models, the variables represent concentrations, population numbers of bacteria or cells or, in general, measures that are nonnegative. Solutions of singular positive systems cannot be solved via the well-established methods for general linear systems. The main reason is that the states of singular positive systems are defined on cones rather than in the whole space. Although many fundamental issues have been well investigated for a class of positive linear systems [9–13], they have not been sufficiently investigated for positive linear implicit difference delay equations. The stability analysis of positive linear implicit difference systems have been considered in [14–18], however, for the system either without delays or with constant delays. In these references, the stability results were obtained by using Lyapunov functional method and the conditions are presented in terms of solutions of some linear matrix inequalities (LMIs).

In this paper, we present some generalizations of exponential stability for the case of positive linear implicit difference systems with time-varying delay. The main contribution of this paper lies in two aspects. First, a novel lemma is proved which not only identifies some specific properties of the decomposed systems, but also enables us to derive new criteria for positivity and stability of the system with interval time-varying delay without using Lyapunov function method. Second, based on the singular value decomposition and mathematical induction approach, delay-dependent necessary and sufficient conditions for the positivity and exponential stability of the systems are derived. Also, numerical examples are presented to illustrate the effectiveness of our results.

The paper is organized as follows. In Section 2, necessary preliminaries are presented and some lemmas are provided. Section 3 proposes necessary and sufficient conditions for the positivity of linear singular difference positive systems with time-varying delay. In Section 4, the corresponding problem is treated for such systems with necessary and sufficient conditions for the exponential stability. Numerical examples are given in these sections to show the effectiveness of the proposed results.

2. Preliminaries

The following notation will be used in this paper. R^n denotes the Euclidean n-dimensional space with the vector norm $\|x\|$; $R^n_{0,+}$ denotes the space of all nonnegative vectors in R^n; $R^{m\times n}$ denotes the set of all real $(m \times n)$ matrices; $\mathbb{N}$ denotes the set of nonnegative integers. If $x = (x_1, x_2, ..., x_n)^T \in R^n$ and $B = (b_{ij}) \in R^{l\times q}$, we define