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Recovery of eigenvectors of rational matrix functions from Fiedler-like linearizations

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ABSTRACT

Linearization is a standard method often used when dealing with matrix polynomials. Recently, the concept of linearization has been extended to rational matrix functions and Fiedler-like matrix pencils for rational matrix functions have been constructed. A linearization $\mathbb{L}(\lambda)$ of a rational matrix function $G(\lambda)$ does not necessarily guarantee a simple way of recovering eigenvectors of $G(\lambda)$ from those of $\mathbb{L}(\lambda)$. We show that Fiedler-like pencils of $G(\lambda)$ allow an easy operation free recovery of eigenvectors of $G(\lambda)$, that is, eigenvectors of $G(\lambda)$ are recovered from eigenvectors of Fiedler-like pencils of $G(\lambda)$ without performing any arithmetic operations. We also consider Fiedler-like pencils of the Rosenbrock system polynomial $\mathcal{S}(\lambda)$ associated with an LTI system Σ in state-space form (SSF) and show that the Fiedler-like pencils allow operation free recovery of eigenvectors of $\mathcal{S}(\lambda)$. The eigenvectors of $\mathcal{S}(\lambda)$ are the invariant zero directions of the LTI system Σ.

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1. Introduction

Let $P(\lambda)$ be an $n \times n$ matrix polynomial (regular or singular) of degree m. Then an $mn \times mn$ matrix pencil $L(\lambda) := A + \lambda B$ is said to be a *linearization* [5,7] of $P(\lambda)$ if there are $mn \times mn$ unimodular matrix polynomials $U(\lambda)$ and $V(\lambda)$ such that $U(\lambda)L(\lambda)V(\lambda) = \text{diag}(I_{(m-1)n}, \ P(\lambda))$ for all $\lambda \in \mathbb{C}$, where I_k denotes the $k \times k$ identity matrix. Linearization is a standard technique often used when dealing with matrix polynomials especially for solving polynomial eigenvalue problems, see [5,7,2,4] and references therein.

Zeros (eigenvalues) and poles of rational matrix functions play an important role in Linear Systems Theory [6,9,11] as well as in many other applications such as in acoustic emissions of high speed trains, calculations of quantum dots, free vibration of plates with elastically attached masses, vibrations of fluid–solid structures, to name only a few, see [8,12,13,10].

Linearizations of rational matrix functions have been introduced recently in [1,3] via matrix-fraction descriptions (MFD) of rational matrix functions. Let $G(\lambda)$ be an $n \times n$ rational matrix function and let $G(\lambda) = N(\lambda)D(\lambda)^{-1}$ be a *right coprime* MFD of $G(\lambda)$, where $N(\lambda)$ and $D(\lambda)$ are matrix polynomials with $D(\lambda)$ being regular. Then the *zero structure* of $G(\lambda)$ is the same as the *eigenstructure* of $N(\lambda)$ and the *pole structure* of $G(\lambda)$ is the same as the *eigenstructure* of $D(\lambda)$, see [6].

Definition 1.1 *(Linearization, [1]).* Let $G(\lambda)$ be an $n \times n$ rational matrix function (regular or singular) and let $G(\lambda) = N(\lambda)D(\lambda)^{-1}$ be a *right coprime* MFD of $G(\lambda)$. Set $r := \deg(\det(D(\lambda)))$. Then a matrix pencil $\mathbb{L}(\lambda)$ of the form

$$\mathbb{L}(\lambda) := \left[\begin{array}{c|c} X + \lambda Y & \mathcal{C} \\ \hline \mathcal{B} & A + \lambda E \end{array}\right] \tag{1.1}$$

is said to be a linearization of $G(\lambda)$ provided that $\mathbb{L}(\lambda)$ is a linearization of $N(\lambda)$ and $A + \lambda E$ is a linearization of $D(\lambda)$, where E is an $r \times r$ nonsingular matrix and the pencil $X + \lambda Y$ and the matrices $\mathcal{B}$ and $\mathcal{C}$ are of appropriate dimensions.

Thus the zeros and the poles of $G(\lambda)$ can be computed by solving the twin generalized eigenvalue problems $\mathbb{L}(\lambda)u = 0$ and $(A + \lambda E)v = 0$. Our main aim in this paper is to recover left and right eigenvectors of $G(\lambda)$ from those of $\mathbb{L}(\lambda)$ when $G(\lambda)$ is regular. The nonzero vectors u and v are said to be left and right eigenvectors of $G(\lambda)$ corresponding to an eigenvalue λ provided that $u^T G(\lambda) = 0$ and $G(\lambda)v = 0$.

The Fiedler-like pencils of $G(\lambda)$ have been constructed in [1,3] by considering a *realization* [6] of $G(\lambda)$ of the form

$$G(\lambda) = \sum_{j=0}^{m} \lambda^j A_j + C(\lambda E - A)^{-1}B =: P(\lambda) + C(\lambda E - A)^{-1}B, \tag{1.2}$$