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Laplacian state transfer in coronas

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We prove that the corona product of two graphs has no Laplacian perfect state transfer whenever the first graph has at least two vertices. This complements a result of Coutinho and Liu who showed that no tree of size greater than two has Laplacian perfect state transfer. In contrast, we prove that the corona product of two graphs exhibits Laplacian pretty good state transfer, under some mild conditions. This provides the first known examples of families of graphs with Laplacian pretty good state transfer. Our result extends the work of Fan and Godsil on double stars to the Laplacian setting. Moreover, we also show that the corona product of any cocktail party graph with a single vertex graph has Laplacian pretty good state transfer, even though odd cocktail party graphs have no perfect state transfer.

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1. Introduction

Given a graph G and a symmetric matrix M associated with G, the continuous-time quantum walk on G relative to M is given by the unitary matrix

$$U(t) := \exp(-itM). \tag{1}$$

This notion was introduced by Farhi and Gutmann [11] as a paradigm to design efficient quantum algorithms. Physically, this also represents the evolution of a quantum spin system. This interesting connection was explored in the works of Bose [3] and Christandl et al. [6,5]. Furthermore, as pointed out by Bose et al. [4], there are two different matrices M of interest. In the so-called XY model, M is the adjacency matrix of G; in the XYZ model, M is the Laplacian of G. For details on these physical models, see Bose et al. [4] for a derivation. Note that if G is regular, these quantum walks differ only by complex conjugation and a phase factor.

From the physical standpoint, quantum walks relative to the adjacency matrix and the Laplacian are equally important. However, the current literature has focused mostly on quantum walks relative to the adjacency matrix. In this paper, we investigate continuous-time quantum walks relative to the Laplacian.

We are interested in the phenomenon of state transfer, which models the routing of information between particles in the associated spin system. This was the original motivation of the work by Bose [3]. A graph G is said to have perfect state transfer between vertices u and v if there is a time t such that

$$|\exp(-itM)_{uv}|^2 = 1. \tag{2}$$

Physically, this means that the probability of state transfer between vertices u and v is unity. We will refer to the matrix entry $\exp(-itM)_{uv}$ as the transition element between the vertices u and v.

There are several infinite families of graphs known to have perfect state transfer. This includes hypercubes [5], some families of distance-regular graphs [8], complete graphs with a missing edge [4], and some joins [1]. However, recently it has become clear that perfect state transfer is rare. In the adjacency matrix case, Godsil showed that there are at most finitely many graphs with a given maximum valency with perfect state transfer [14], while, in the Laplacian case, Coutinho and Liu showed that there is no perfect state transfer on trees with at least three vertices [9].

Nevertheless, transmission of information in a quantum system may not occur perfectly, but rather with probability that is arbitrarily close to unity. We thus consider a relaxation. A graph G is said to have pretty good state transfer if for each $\epsilon > 0$, there exists a time t such that

$$|\exp(-itM)_{uv}|^2 \geq 1 - \epsilon. \tag{3}$$