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# Miniversal deformations of pairs of skew-symmetric matrices under congruence ☆

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ABSTRACT

Miniversal deformations for pairs of skew-symmetric matrices under congruence are constructed. To be precise, for each such a pair $(A, B)$ we provide a normal form with a minimal number of independent parameters to which all pairs of skew-symmetric matrices $(\widetilde{A}, \widetilde{B})$, close to $(A, B)$ can be reduced by congruence transformation which smoothly depends on the entries of the matrices in the pair $(\widetilde{A}, \widetilde{B})$. An upper bound on the distance from such a miniversal deformation to $(A, B)$ is derived too. We also present an example of using miniversal deformations for analyzing changes in the canonical structure information (i.e. eigenvalues and minimal indices) of skew-symmetric matrix pairs under perturbations.



## 1. Introduction

Canonical forms of matrices and matrix pencils, e.g., Jordan and Kronekher canonical forms, are well known and studied with various purposes but the reductions to these forms are unstable operations: both the corresponding canonical forms and the reduction transformations depend discontinuously on the entries of an original matrix or matrix

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pencil. Therefore, V.I. Arnold introduced a normal form, with the minimal number of independent parameters, to which an arbitrary family of matrices $\tilde{A}$ close to a given matrix $A$ can be reduced by similarity transformations smoothly depending on the entries of $\tilde{A}$. He called such a normal form a miniversal deformation of $A$. Now the notion of miniversal deformations has been extended to matrices with respect to congruence [14] and *congruence [15], matrix pencils with respect to strict equivalence [19,23] and congruence [11], etc. (more detailed list is given in the introduction of [15]).

Miniversal deformations can help us to construct stratifications, i.e., closure hierarchies, [13,20,21] of orbits and bundles. These stratifications are the graphs that show which canonical forms the matrices (or matrix pencils) may have in an arbitrarily small neighborhood of a given matrix (or matrix pencil). In particular, the stratifications show how the eigenvalues may coalesce or split apart, appear or disappear. Both the stratifications and miniversal deformations may be useful when the matrices arise as a result of measures and their entries are known with errors, see [27,30] for some applications in control and stability theory.

The questions related to eigenvalues and another canonical information for the pencils $A - sB$, where $A = \pm A^T$ and $B = \pm B^T$, or $A = \pm A^*$ and $B = \pm B^*$, dragged some attention over time and, especially, recently, e.g., see the following papers on canonical forms [35,37], codimension computations [8,9,17,18], low rank perturbations [4], miniversal deformations [11,14,23], partial [13,22] and general [16] stratification results, staircase forms [5,7]. Such pencils also appear as the structure preserving linearizations of the corresponding matrix polynomials [32,33]. In particular, the papers [4,16,17,35,37] deal with skew-symmetric matrix pencils, i.e. $A - sB$, where $A = -A^T$ and $B = -B^T$, and [33] deals with skew-symmetric matrix polynomials. Skew-symmetric matrix pencils appear in multisymplectic partial differential equations [6], systems with bi-Hamiltonian structure [34], as well as in the design of a passive velocity field controller [28]. Recall that, an $n \times n$ skew-symmetric matrix pencil $A - sB$ is called congruent to $C - sD$ if and only if there is a non-singular matrix $S$ such that $S^T AS = C$ and $S^T BS = D$. The set of matrix pencils congruent to a skew-symmetric matrix pencil $A-sB$ is called a congruence orbit of $A-sB$.

In this paper, we derive the miniversal deformations of skew-symmetric matrix pencils under congruence and bound the distance from these deformations to unperturbed matrix pencils in terms of the norm of the perturbations. The number of independent parameters in the miniversal deformations is equal to the codimensions of the congruence orbits of skew-symmetric matrix pencils (obtained independently in [17]). The Matlab functions for computing these codimensions were developed [12] and added to the Matrix Canonical Structure (MCS) Toolbox [25]. Example 2.1 shows how the miniversal deformations from Theorem 2.1 can be used for the investigation of the possible changes of the canonical structure information.

The rest of the paper is organized as follows. In Section 2, we present the main theorems, i.e., we construct miniversal deformations of skew-symmetric matrix pencils and prove an upper bound on the distance between a skew-symmetric matrix pencil and its miniversal deformation. In Section 3, we describe the method of constructing