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Weak-local triple derivations on C*-algebras and JB*-triples ☆

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ABSTRACT

We prove that every weak-local triple derivation on a JB*-triple E (i.e. a linear map $T : E \to E$ such that for each $\phi \in E^*$ and each $a \in E$, there exists a triple derivation $\delta_{a,\phi} : E \to E$, depending on ϕ and a, such that $\phi T(a) = \phi\delta_{a,\phi}(a)$) is a (continuous) triple derivation. We also prove that conditions

($h1$) $\{a, T(b), c\} = 0$ for every a, b, c in E with $a, c \perp b$;
($h2$) $P_2(e)T(a) = -Q(e)T(a)$ for every norm-one element a in E, and every tripotent e in E^{**} such that $e \leq s(a)$ in $E_2^{**}(e)$, where $s(a)$ is the support tripotent of a in E^{**},

are necessary and sufficient to show that a linear map T on a JB*-triple E is a triple derivation.

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1. Introduction

A *triple derivation* on a JB*-triple E is a linear mapping $\delta : E \to E$ satisfying the following Leibniz' rule

$$\delta\{a, b, c\} = \{\delta(a), b, c\} + \{a, \delta(b), c\} + \{a, b, \delta(c)\}, \tag{1}$$

for every $a, b, c \in E$. T. Barton and Y. Friedman proved, in [1, Corollary 2.2], that the geometric structure of JB*-triples assures that every triple derivation on a JB*-triple is continuous.

Motivated by the different studies on local (associative) derivations on C*-algebras (compare [18,22,17]), M. Mackey introduced and presented, in [20], the first study about local triple derivations on JB*-triples. We recall that a *local triple derivation* on a JB*-triple E is a linear map $T : E \to E$ such that for each a in E there exists a triple derivation $\delta_a : E \to E$, depending on a, satisfying $T(a) = \delta_a(a)$. It is due to Mackey that every continuous local triple on a JBW*-triple (i.e. a JB*-triple which is also a dual Banach space) is a triple derivation (see [20, Theorem 5.11]). The first and third author of this note, in collaboration with F.J. Fernández-Polo, established in [7] that every local triple derivation on a JB*-triple is continuous and a triple derivation.

In the setting of C*-algebras, B.A. Essaleh, M.I. Ramírez, and the third author of this note explore the notions of weak-local derivation and weak*-local derivation on C*-algebras and von Neumann algebras, respectively (see [10,11]). Going back in history, we remind that, according to Kadison's definition, a *local derivation* from a C*-algebra A into a Banach A-bimodule, X, is a linear map $T : A \to X$ such that for each $a \in A$ there exits a derivation $D_a : A \to X$, depending on a, satisfying $T(a) = D_a(a)$. B.E. Johnson proved in [17] that every local derivation from a C*-algebra A into a Banach A-bimodule is continuous and a derivation. Following [10], a linear mapping $T : A \to X$ is called a *weak-local derivation* if for each $\phi \in X^*$ and each $a \in A$, there exists a derivation $D_{a,\phi} : A \to X$, depending on ϕ and a, such that $\phi T(a) = \phi D_{a,\phi}(a)$. It is shown in [10, Theorems 2.1 and 3.4] (see also [11]) that every weak-local derivation on a C*-algebra is continuous and a derivation. Similarly, if W is a von Neumann algebra (i.e. a C*-algebra which is also a dual Banach space) a *weak*-local derivation* on W is a linear map $T : W \to W$ such that for each $\phi \in W_*$ and each $a \in W$, there exists a derivation $D_{a,\phi} : W \to W$, depending on ϕ and a, such that $\phi T(a) = \phi D_{a,\phi}(a)$. Weak*-local derivations on a von Neumann algebra are automatically continuous and derivations (see [10, Theorems 2.8 and 3.1]).

In the wider setting of JB*-triples, weak-local triple derivations seem a natural notion to explore in this line. We shall say that a linear map T on a JB*-triple E is a *weak-local triple derivation* if for each $\phi \in E^*$ and each $a \in E$, there exists a triple derivation $\delta_{a,\phi} : E \to E$, depending on ϕ and a, such that $\phi T(a) = \phi\delta_{a,\phi}(a)$. *Weak*-local triple derivations* on a JBW*-triple are similarly defined.