

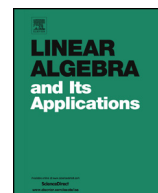


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# Linear Algebra and its Applications

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## Weak-local triple derivations on $C^*$ -algebras and $JB^*$ -triples <sup>☆</sup>



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### ARTICLE INFO

#### Article history:

Received 15 April 2016

Accepted 24 June 2016

Available online 29 June 2016

Submitted by P. Semrl

#### MSC:

47B47

46L57

17C65

47C15

46L05

46L08

#### Keywords:

$C^*$ -algebra

$JB^*$ -triple

Triple derivation

Weak-local triple derivation

Weak-local derivation

### ABSTRACT

We prove that every weak-local triple derivation on a  $JB^*$ -triple  $E$  (i.e. a linear map  $T : E \rightarrow E$  such that for each  $\phi \in E^*$  and each  $a \in E$ , there exists a triple derivation  $\delta_{a,\phi} : E \rightarrow E$ , depending on  $\phi$  and  $a$ , such that  $\phi T(a) = \phi \delta_{a,\phi}(a)$ ) is a (continuous) triple derivation. We also prove that conditions

- (h1)  $\{a, T(b), c\} = 0$  for every  $a, b, c$  in  $E$  with  $a, c \perp b$ ;
- (h2)  $P_2(e)T(a) = -Q(e)T(a)$  for every norm-one element  $a$  in  $E$ , and every tripotent  $e$  in  $E^{**}$  such that  $e \leq s(a)$  in  $E_2^{**}(e)$ , where  $s(a)$  is the support tripotent of  $a$  in  $E^{**}$ ,

are necessary and sufficient to show that a linear map  $T$  on a  $JB^*$ -triple  $E$  is a triple derivation.

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<sup>☆</sup> Authors partially supported by the Spanish Ministry of Economy and Competitiveness and European Regional Development Fund project no. MTM2014-58984-P and Junta de Andalucía grants FQM375 and FQM290.

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## 1. Introduction

A *triple derivation* on a JB\*-triple  $E$  is a linear mapping  $\delta : E \rightarrow E$  satisfying the following Leibniz' rule

$$\delta\{a, b, c\} = \{\delta(a), b, c\} + \{a, \delta(b), c\} + \{a, b, \delta(c)\}, \quad (1)$$

for every  $a, b, c \in E$ . T. Barton and Y. Friedman proved, in [1, Corollary 2.2], that the geometric structure of JB\*-triples assures that every triple derivation on a JB\*-triple is continuous.

Motivated by the different studies on local (associative) derivations on C\*-algebras (compare [18,22,17]), M. Mackey introduced and presented, in [20], the first study about local triple derivations on JB\*-triples. We recall that a *local triple derivation* on a JB\*-triple  $E$  is a linear map  $T : E \rightarrow E$  such that for each  $a$  in  $E$  there exists a triple derivation  $\delta_a : E \rightarrow E$ , depending on  $a$ , satisfying  $T(a) = \delta_a(a)$ . It is due to Mackey that every continuous local triple on a JBW\*-triple (i.e. a JB\*-triple which is also a dual Banach space) is a triple derivation (see [20, Theorem 5.11]). The first and third author of this note, in collaboration with F.J. Fernández-Polo, established in [7] that every local triple derivation on a JB\*-triple is continuous and a triple derivation.

In the setting of C\*-algebras, B.A. Essaleh, M.I. Ramírez, and the third author of this note explore the notions of weak-local derivation and weak\*-local derivation on C\*-algebras and von Neumann algebras, respectively (see [10,11]). Going back in history, we remind that, according to Kadison's definition, a *local derivation* from a C\*-algebra  $A$  into a Banach  $A$ -bimodule,  $X$ , is a linear map  $T : A \rightarrow X$  such that for each  $a \in A$  there exists a derivation  $D_a : A \rightarrow X$ , depending on  $a$ , satisfying  $T(a) = D_a(a)$ . B.E. Johnson proved in [17] that every local derivation from a C\*-algebra  $A$  into a Banach  $A$ -bimodule is continuous and a derivation. Following [10], a linear mapping  $T : A \rightarrow X$  is called a *weak-local derivation* if for each  $\phi \in X^*$  and each  $a \in A$ , there exists a derivation  $D_{a,\phi} : A \rightarrow X$ , depending on  $\phi$  and  $a$ , such that  $\phi T(a) = \phi D_{a,\phi}(a)$ . It is shown in [10, Theorems 2.1 and 3.4] (see also [11]) that every weak-local derivation on a C\*-algebra is continuous and a derivation. Similarly, if  $W$  is a von Neumann algebra (i.e. a C\*-algebra which is also a dual Banach space) a *weak\*-local derivation* on  $W$  is a linear map  $T : W \rightarrow W$  such that for each  $\phi \in W_*$  and each  $a \in W$ , there exists a derivation  $D_{a,\phi} : W \rightarrow W$ , depending on  $\phi$  and  $a$ , such that  $\phi T(a) = \phi D_{a,\phi}(a)$ . Weak\*-local derivations on a von Neumann algebra are automatically continuous and derivations (see [10, Theorems 2.8 and 3.1]).

In the wider setting of JB\*-triples, weak-local triple derivations seem a natural notion to explore in this line. We shall say that a linear map  $T$  on a JB\*-triple  $E$  is a *weak-local triple derivation* if for each  $\phi \in E^*$  and each  $a \in E$ , there exists a triple derivation  $\delta_{a,\phi} : E \rightarrow E$ , depending on  $\phi$  and  $a$ , such that  $\phi T(a) = \phi \delta_{a,\phi}(a)$ . *Weak\*-local triple derivations* on a JBW\*-triple are similarly defined.

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