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Extremizing first eigenvalue of 3-colored digraphs made with given blocks

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ABSTRACT

This article describes the structure of the non-singular connected 3-colored digraphs extremizing the first eigenvalue among all such graphs made with some given blocks. As an application we determine the non-singular unicyclic 3-colored digraph, which minimizes the first eigenvalue over all such graphs with fixed order and girth.

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1. Introduction

The notion of weighted directed graph was introduced in [2]. A *weighted directed graph* is a directed graph with a simple underlying undirected graph and edges having complex weights of unit modulus (see [2]). Thus in a weighted directed graph both the edges (i, j) and (j, i) cannot be present simultaneously. Furthermore, the presence of an edge (i, j)

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of weight w is as good as the presence of the edge (j, i) of weight $\overline{\mathsf{w}}$, where $\overline{\mathsf{w}}$ is the complex conjugate of w. Weighted directed graphs with edge weights of unit modulus are also known as $\mathbb{T}$-*gain graphs* (see [16]), where $\mathbb{T}$ stands for the complex numbers of unit modulus. In this article, w_{ij} denotes the weight of an edge (i, j).

Let G be a weighted directed graph on vertices $1, 2, \ldots, n$. At times, we use $V(G)$ and $E(G)$ to denote the set of vertices and the set of edges of G, respectively. For convenience we use '$i \sim j$' to mean '$(i, j) \in E(G)$ or $(j, i) \in E(G)$'. Bapat, Kalita and Pati defined the adjacency and Laplacian matrix of the weighted directed graphs in [2]. The *adjacency matrix*, $A(G) = [a_{ij}]$ of G is an $n \times n$ matrix with entries $a_{ij} = \mathsf{w}_{ij}$ or $\overline{\mathsf{w}}_{ji}$ or 0, depending on whether $(i, j) \in E(G)$ or $(j, i) \in E(G)$ or otherwise, respectively. The *degree* $d(i)$ of a vertex i in G is the number of edges incident with i. The *Laplacian matrix* $L(G)$ of G is defined as $L(G) = D(G) - A(G)$, where $D(G)$ is the diagonal matrix with $d(i)$ as the i-th diagonal entry. In particular, if weights of all the edges in G are 1, then $L(G)$ coincides with the usual Laplacian matrix of an unweighted undirected graph. Thus, henceforth, a weighted directed graph with all the edges having weight 1 is said to be an *unweighted undirected* graph. Similarly, if weights of all the edges in G are -1, then $L(G)$ coincides with the well studied signless Laplacian (see for example, Cvetkovic, Rowlinson and Simic [4]) of an undirected graph G. One can also see that, if weights of all the edges in G are 1 or -1, then (*viewing the edges of weight* 1 *as directed and the edges with weight* -1 *as undirected*) $L(G)$ coincides with the Laplacian matrix of a mixed graph (see [1,10]). Thus study of the Laplacian matrices of weighted directed graphs includes the study of Laplacian matrices of unweighted undirected graphs, mixed graphs and the study of the signless Laplacian matrices as special cases. In [2], the authors proved that $L(G)$ is Hermitian and positive semidefinite. Furthermore, unlike the Laplacian matrix of unweighted undirected graph, the Laplacian matrix of weighted directed graph is sometimes **nonsingular** (see [2]). A connected weighted directed graph is said to be *singular* (resp. *nonsingular*) if its corresponding Laplacian matrix is singular (resp. nonsingular). Several characterizations of singularity of weighted directed graphs can be found in [2].

In this article, we shall restrict our attention to the weighted directed graphs with weights from $\{\pm 1, \pm \mathrm{i}\}$, where $\mathrm{i} = \sqrt{-1}$. For the purpose of studying the Laplacian matrix, one can replace each edge (a, b) of weight $-\mathrm{i}$ with an edge (b, a) of weight i, as it will not change the Laplacian matrix. In view of this, such a graph may be thought of having edges of weights ± 1 and i only. It is very convenient to view the edges of weight 1 as red edges, edges of weight -1 as blue edges and edges of weight i as green edges. We shall be referring to them as *3-colored* digraphs (see [2]). At times, while referring to the colors of the edges, we shall use red $(+1)$, blue (-1) and green (i), to remind the reader of their weights. Note that the orientations of the red $(+1)$ and blue (-1) edges are inconsequential for the study of the Laplacian matrix. Hence the orientations of these edges are not indicated in the figures. Our coloring of edges has nothing to do with graph coloring. We refer the reader to [2,5,6], for more backgrounds on 3-colored digraphs. In defining subgraph, walk, path, block, component, connectedness, and degree of a vertex

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