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Zero sum sign-central matrices and applications

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ABSTRACT

A matrix with a nonzero nonnegative vector in its null space is called *central*. We study classes of central matrices having zero column sums. The study is motivated by an engineering application concerning induction heating where central matrices provide a way to control the energy flow over time. A (± 1)-matrix A is called a *ZSC-matrix* (zero sum sign-central) if each matrix with the same sign pattern as A and having zero column sums is central. We establish several classes of ZSC-matrices, and give separate sufficient and necessary conditions for a matrix to be ZSC. Moreover, we give algorithms for finding central matrices that are used for power control in induction heating, and illustrate these by some numerical examples.

1. Introduction

Let $A = [a_{ij}]$ be an $m \times n$ matrix. A is called *central* if there is a nonzero nonnegative vector in its null space. The row space of A is denoted by Row A and its null space is

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denoted by Nul A. Let csum A denote the column sum vector of A, so its jth component is $\sum_{i=1}^{m} a_{ij}$ ($j \leq n$). Similarly, rsum A is the row sum vector of A. The sign matrix of a matrix A, denoted sign A, is the matrix obtained by replacing each entry in A by its sign (+, − or 0). The qualitative class $\mathcal{Q}(A)$ of A consists of all matrices with the same sign matrix as A. A matrix A is called *sign-central* if each matrix in the qualitative class of A is central. Sign-central matrices were introduced and studied in [1], and a characterization was found. This matrix class is also treated in [2] along with qualitative matrix theory in general. The related notion of strict sign-central matrices was studied in [3]. For related combinatorial properties of matrices, we refer to [4,5].

Throughout the paper, we are concerned with central matrices, and a new notion, closely related to sign-centrality. We call a (±1)-matrix A a *ZSC-matrix* (zero sum sign-central) if each matrix with the same sign pattern as A and having zero column sums is central. The investigation is motivated by an engineering problem where one wants to perform efficient power control in induction heating. It turns out that central matrices with zero column sums play an important role for such control. We outline this application in Section 4. Since there is a great uncertainty in the quantitative data of this application, qualitative matrix theory is used as a key tool for finding central matrices to perform the control. Qualitative matrix theory is rooted in economics, and it is interesting that it also plays a role in an important industrial engineering application, as discussed in this paper.

The paper is organized as follows. In Section 2 central matrices are characterized and a connection to the reduced echelon form of matrix is discussed. Section 3 contains an analysis of ZSC matrices. It gives separate sufficient and necessary conditions for a matrix to be ZSC. Several classes of ZSC matrices are established. These matrix classes have a combinatorial structure. In Section 4 we describe the engineering application, and algorithms based on results from Section 3 for power control. Some numerical examples are also given.

We treat vectors in $\mathbb{R}^n$ as column vectors and identify these with corresponding n-tuples. $M_{m,n}$ denotes the set of all real $m \times n$ matrices, and when $m = n$ we just write M_n. O denotes the zero matrix or vector. I_n (or just I) is the identity matrix of order n. We let e_i denote the ith unit (coordinate) vector in $\mathbb{R}^n$, and e denotes the all ones vector. A vector $x = (x_1, x_2, \ldots, x_n)$ is *nonnegative* if each component x_i is nonnegative. Similarly, we define *positive*, *negative*, and *nonpositive* vectors. The kth smallest component of the vector x is denoted by $x_{(k)}$.

2. Central matrices

We first study the class of central matrices. A characterization of this class may be derived using separation of convex sets [9], or by Farkas' lemma/duality. Note that if a matrix contains a zero column, then it is trivially central. Thus, in this section we restrict the attention to matrices with no zero column (unless otherwise stated).