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On the conditioning of factors in the SR decomposition

H. Faßbender [a], M. Rozložník [b,*,1]

[a] Institut Computational Mathematics, AG Numerik, Technische Universität Braunschweig, Pockelsstr. 14, 38106 Braunschweig, Germany
[b] Institute of Computer Science, Czech Academy of Sciences, Pod vodárenskou věží 2, CZ-182 07 Prague 8, Czech Republic

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ABSTRACT

Almost every nonsingular matrix $A \in \mathbb{R}^{2m,2m}$ can be decomposed into the product of a symplectic matrix S and an upper J-triangular matrix R. This decomposition is not unique. In this paper we analyze the freedom of choice in the symplectic and the upper J-triangular factors and review several existing suggestions on how to choose the free parameters in the SR decomposition. In particular we consider two choices leading to the minimization of the condition number of the diagonal blocks in the upper J-triangular factor and to the minimization of the conditioning of the corresponding blocks in the symplectic factor. We develop bounds for the extremal singular values of the whole upper J-triangular factor and the whole symplectic factor in terms of the spectral properties of even-dimensioned principal submatrices of the skew-symmetric matrix associated with the SR decomposition. The theoretical results are illustrated on two small examples.

* Corresponding author.
E-mail addresses: h.fassbender@tu-braunschweig.de (H. Faßbender), miro@cs.cas.cz (M. Rozložník).

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1. Introduction

For each natural number m we define the skew-symmetric matrix

$$J_{2m} = \begin{pmatrix} 0 & I_m \\ -I_m & 0 \end{pmatrix} \in \mathbb{R}^{2m,2m},$$

where $I_m \in \mathbb{R}^{m,m}$ denotes the identity matrix of the order m. It is clear that J_{2m} is nonsingular with $J_{2m}^{-1} = J_{2m}^T = -J_{2m}$. If $I_{2m} = [e_1, \dots e_{2m}]$ is the identity matrix of order $2m$ we define a permutation matrix $P_{2m} \in \mathbb{R}^{2m,2m}$ as

$$P_{2m} = [e_1, e_3, \dots, e_{2m-1}, e_2, e_4, \dots, e_{2m}].$$

It follows that $P_{2m}^{-1} = P_{2m}^T = [e_1, e_{m+1}, e_2, e_{m+2}, \dots, e_m, e_{2m}]$. Using the permutation matrix P_{2m}, the matrix J_{2m} can be permuted to the block diagonal matrix $\hat{J}_{2m} \in \mathbb{R}^{2m,2m}$ such that

$$\hat{J}_{2m} = P_{2m} J_{2m} P_{2m}^T = \mathrm{diag}(\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \dots, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}).$$

Definition 1. A real square matrix $S_{2m} \in \mathbb{R}^{2m,2m}$ is a symplectic matrix if $S_{2m}^T J_{2m} S_{2m} = J_{2m}$. Similarly, a real rectangular matrix $S_{2m,2n} \in \mathbb{R}^{2m,2n}$ is called semi-symplectic if $S_{2m,2n}^T J_{2m} S_{2m,2n} = J_{2n}$.

Definition 2. A matrix $R_{2m} = \begin{pmatrix} R_{1,1} & R_{1,2} \\ R_{2,1} & R_{2,2} \end{pmatrix} \in \mathbb{R}^{2m,2m}$ is an upper J-triangular matrix if $R_{1,1}$, $R_{1,2}$, and $R_{2,2} \in \mathbb{R}^{m,m}$ are upper triangular matrices and $R_{2,1} \in \mathbb{R}^{m,m}$ is strictly upper triangular matrix.

Note that if R_{2m} is an upper J-triangular matrix, then the matrix $\hat{R}_{2m} = P_{2m} R_{2m} P_{2m}^T \in \mathbb{R}^{2m,2m}$ is an upper triangular matrix of the form

$$\hat{R}_{2m} = \begin{pmatrix} \hat{\mathrm{R}}_{1,1} & \dots & \hat{\mathrm{R}}_{1,m} \\ 0 & \ddots & \vdots \\ 0 & 0 & \hat{\mathrm{R}}_{m,m} \end{pmatrix},$$

where $\hat{\mathrm{R}}_{i,j} \in \mathbb{R}^{2,2}$ for $i = 1, \dots j$; $j = 1, \dots, m$ and $\hat{\mathrm{R}}_{j,j}$ is upper triangular for $j = 1, \dots, m$.

The question whether a square matrix of order $2m$ can be decomposed into the product of a symplectic matrix and an upper J-triangular matrix is answered in the following theorem.