

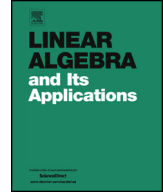


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Row sums and alternating sums of Riordan arrays



Tian-Xiao He^{a,*}, Louis W. Shapiro^b

^a Department of Mathematics, Illinois Wesleyan University, Bloomington,
IL 61702-2900, USA

^b Department of Mathematics, Howard University, Washington, DC 20059, USA

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ABSTRACT

Here we use row sum generating functions and alternating sum generating functions to characterize Riordan arrays and subgroups of the Riordan group. Numerous applications and examples are presented which include the construction of Girard–Waring type identities. We also show the extensions to weighted sum (generating) functions, called the expected value (generating) functions of Riordan arrays.

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* Corresponding author.

E-mail address: the@iwu.edu (T.-X. He).

1. Introduction

Riordan arrays are infinite, lower triangular matrices defined by two generating functions. They form a group, called the Riordan group (see Shapiro et al. [15]). More formally, consider the set of formal power series (f.p.s.) $\mathcal{F} = \mathbb{R}[[t]]$; the order of $f(t) \in \mathcal{F}$, $f(t) = \sum_{k=0}^{\infty} f_k t^k$ ($f_k \in \mathbb{R}$), is the minimal number $r \in \mathbb{N}$ such that $f_r \neq 0$. The set of formal power series of order r is denoted by $\mathcal{F}_r$. It is known that $\mathcal{F}_0$ is the set of invertible f.p.s. and $\mathcal{F}_1$ is the set of compositionally invertible f.p.s., that is, the f.p.s. $f(t)$ for which the compositional inverse $\bar{f}(t)$ exists such that $f(\bar{f}(t)) = \bar{f}(f(t)) = t$. Let $d(t) \in \mathcal{F}_0$ and $h(t) \in \mathcal{F}_1$; the pair $(d(t), h(t))$ defines the (proper) Riordan array $D = (d_{n,k})_{n,k \geq 0} = (d(t), h(t))$, where

$$d_{n,k} = [t^n]d(t)h(t)^k \quad (1)$$

or, in other words, $d(t)h(t)^k$ is the generating function for the entries of column k .

Let $[f_0, f_1, f_2, \dots]^T$ be a column vector with $f(t) = \sum_{n \geq 0} f_n t^n$. It is convenient to switch freely between a sequence, a sequence written as a column vector, and the ordinary generating function for that sequence. We then have the fundamental theorem of Riordan arrays

$$(d(t), h(t)) [f_0, f_1, f_2, \dots]^T = (d(t), h(t))f(t) = d(t)f(h(t)). \quad (2)$$

It follows quickly that the usual row-by-column product of two Riordan arrays is also a Riordan array:

$$(d_1(t), h_1(t)) * (d_2(t), h_2(t)) = (d_1(t)d_2(h_1(t)), h_2(h_1(t))). \quad (3)$$

The Riordan array $I = (1, t)$ is everywhere 0 except for all 1's on the main diagonal; it can be easily proved that I acts as an identity for this product, that is, $(1, t) * (d(t), h(t)) = (d(t), h(t)) * (1, t) = (d(t), h(t))$. Let $(d(t), h(t))$ be a Riordan array. Then its inverse is

$$(d(t), h(t))^{-1} = \left(\frac{1}{d(\bar{h}(t))}, \bar{h}(t) \right), \quad (4)$$

where $\bar{h}(t)$ is the compositional inverse of $h(t)$. In this way, the set $\mathcal{R}$ of proper Riordan arrays forms a group (see [15]).

Multiplying a matrix by the column vector $[1, 1, 1, \dots]^T$ yields the column of row sums which we denote as R^+ . Since $f(t) = 1/(1-t)$ is the corresponding generating function, then (2) presents the generating function R^+ of the row sum sequence of the Riordan array $(d(t), h(t))$ (see [14]), i.e.,

$$R^+(t) := (d(t), h(t)) \frac{1}{1-t} = \frac{d(t)}{1-h(t)}. \quad (5)$$

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