Linear Algebra and its Applications 500 (2016) 52–62

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Cospectral digraphs from locally line digraphs

C. Dalfó [a,*], M.A. Fiol [b]

[a] Departament de Matemàtiques, Universitat Politècnica de Catalunya, Barcelona, Catalonia

[b] Barcelona Graduate School of Mathematics, Barcelona, Catalonia

ARTICLE INFO

Article history:
Received 13 November 2015
Accepted 10 March 2016
Available online 19 March 2016
Submitted by R. Brualdi

MSC:
05C20
05C50

Keywords:
Digraph
Adjacency matrix
Spectrum
Cospectral digraph
Diameter
De Bruijn digraph
Kautz digraph

ABSTRACT

A digraph $\Gamma = (V, E)$ is a line digraph when every pair of vertices $u, v \in V$ have either equal or disjoint in-neighborhoods. When this condition only applies for vertices in a given subset (with at least two elements), we say that Γ is a locally line digraph. In this paper we give a new method to obtain a digraph Γ' cospectral with a given locally line digraph Γ with diameter D, where the diameter D' of Γ' is in the interval $[D-1, D+1]$. In particular, when the method is applied to De Bruijn or Kautz digraphs, we obtain cospectral digraphs with the same algebraic properties that characterize the formers.

1. Preliminaries

In this section we recall some basic terminology and simple results concerning digraphs and their spectra. For the concepts and/or results not presented here, we refer the reader

* Corresponding author.
E-mail addresses: cristina.dalfo@upc.edu (C. Dalfó), miguel.angel.fiol@upc.edu (M.A. Fiol).

http://dx.doi.org/10.1016/j.laa.2016.03.014

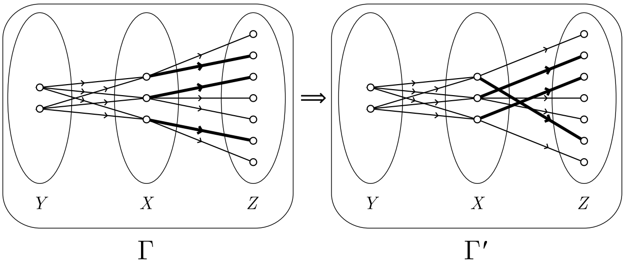

Fig. 1. Scheme of the sets of Theorem 2.1. The arcs that change from Γ to Γ' are represented with a thick line.

to some of the basic textbooks and papers on the subject; for instance, Chartrand and Lesniak [1] and Diestel [3].

Through this paper, $\Gamma = (V, E)$ denotes a digraph, with set of vertices $V = V(\Gamma)$ and set of arcs (or directed edges) $E = E(\Gamma)$, that is strongly connected (namely, every vertex is connected to any other vertex by traversing the arcs in their corresponding direction). An arc from vertex u to vertex v is denoted by either (u, v) or $u \to v$. As usual, we call *loop* an arc from a vertex to itself, $u \to u$, and *digon* to two opposite arcs joining a pair of vertices, $u \rightleftarrows v$. The set of vertices adjacent to and from $v \in V$ is denoted by $\Gamma^-(v)$ and $\Gamma^+(v)$, respectively. Such vertices are referred to as *in-neighbors* and *out-neighbors* of v, respectively. Moreover, $\delta^-(v) = |\Gamma^-(v)|$ and $\delta^+(v) = |\Gamma^+(v)|$ are the *in-degree* and *out-degree* of vertex v, and Γ is *d-regular* when $\delta^+(v) = \delta^-(v) = d$ for any $v \in V$. Similarly, given $U \subset V$, $\Gamma^-(U)$ and $\Gamma^+(U)$ represent the sets of vertices adjacent to and from (the vertices of) U. Given two vertex subsets $X, Y \subset V$, the subset of arcs from X to Y is denoted by $e(X, Y)$.

In the line digraph $L\Gamma$ of a digraph Γ, each vertex represents an arc of Γ, $V(L\Gamma) = \{uv : (u, v) \in E(G)\}$, and a vertex uv is adjacent to a vertex wz when $v = w$, that is, when in Γ the arc (u, v) is adjacent to the arc (w, z): $u \to v(= w) \to z$. By the Heuchenne's condition [9], a digraph Γ is a line digraph if and only if, for every pair of vertices u, v, either $\Gamma^+(u) = \Gamma^+(v)$ or $\Gamma^+(u) \cap \Gamma^+(v) = \emptyset$. Since the line digraph of the converse digraph $\overline{\Gamma}$ (obtained from Γ by reversing the directions of all the arcs) equals the converse of the line digraph, $L\overline{\Gamma} = \overline{L\Gamma}$, the above condition can be restated in terms of the in-neighborhoods $\Gamma^-(u)$ and $\Gamma^-(v)$. In particular, we say that a digraph is a (U-)*locally line digraph* if there is a vertex subset U with at least two elements such that $\Gamma^-(u) = \Gamma^-(v)$ for every $u, v \in U$.

In the case of graphs instead of digraphs, the Godsil–McKay switching given in [8] is a technique to obtain cospectral graphs.

2. Main result

The following result describes the basic transformation of a digraph Γ into another digraph Γ' modifying slightly the walk properties of the former (see Fig. 1).