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Sparse matrix approximations for multigrid methods

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We discuss the application of sparse matrix approximations for two-grid and V-cycle multigrid methods. Sparse approximate inverses can be used as smoothers, further the Galerkin coarse matrix can be sparsified by sparse approximation techniques. Also the projection can be defined by combining sparse approximation with side conditions related to high frequency components. Numerical results are given, which demonstrate the efficiency and accuracy of the proposed strategies.
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1. Introduction

A significant research direction within the scope of this paper is the derivation and development of practicable multigrid algorithms, which operate as direct or approximate direct solvers for linear systems. Multigrid algorithms are iterative methods defined starting from two basic ingredients: the smoother and the projection (the grid transfer operator). Several multigrid methods have been introduced in the literature and have shown to be very effective for different kind of problems, see [20,18]. A multigrid cycle that converges in just one iteration step is considered to be a direct solver. In the case of approximate direct solvers the convergence rate is expected to be very low. In [20, Paragraph A.2.3] there has been proposed a theoretically intriguing result that characterizes direct solvers in general. But with respect to practicability, this scheme is not always feasible, since the computation of the smoother and of the projection includes an explicit matrix inversion. In view of the recursive nature of multigrid this methodology leads to prohibitive numerical costs. We aim at deriving practicable and efficient multigrid algorithms with the aid of the notion of Compact Fourier Analysis (CFA), cf. [13,14]. With respect to the practicability, the appearing matrices should be structured and sparse. The ideal case is when no matrix inversion is needed. It is sensible to consider smoothers and projections given by band matrices. According to the following notion of generating function, the request to work with band smoothers and projections is equivalent to require that they have trigonometric polynomials as generating functions.

The approach analyzed here is based on the well-known connection between Toeplitz matrices and trigonometric functions. A Toeplitz matrix $T_n = T_n(f) = T_n(t_{-n+1}, \ldots, t_{n-1})$ of order n with constant entries on the diagonals, i.e., $[T_n]_{j,k} = t_{j-k}$ for $j, k = 1, \ldots, n$, is associated with the scalar generating function

$$f(x) = \sum_{j=-\infty}^{\infty} t_j \mathrm{e}^{\mathrm{i}jx}, \qquad \mathrm{i} = \sqrt{-1}.$$

In applications corresponding to two-dimensional problems the system matrix is not Toeplitz but rather a block Toeplitz matrices where the blocks themselves are Toeplitz again. In this so-called BTTB-case the generating symbol is given by

$$f(x,y) = \sum_{j,k\in\mathbb{Z}} t_{j,k} \mathrm{e}^{\mathrm{i}jx+\mathrm{i}ky}. \tag{1.1}$$

The extension to higher dimensional problems is straightforward. For more relevant details, cf., e.g., [10,11,13,14] and the references therein.

A fundamental ingredient of CFA is the block symbol, which is a matrix function and is derived as follows. The block symbol in the 2D case can be given by a Fourier series, with coefficients being matrices $F_{j,k}$, in the form

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