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Rank-revealing decomposition of symmetric indefinite matrices via block anti-triangular factorization

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ABSTRACT

We present an algorithm for computing a symmetric rank-revealing decomposition of a symmetric $n \times n$ matrix A, as defined in the work of Hansen & Yalamov [9]: we factorize the original matrix into a product $A = QMQ^T$, with Q orthogonal and M symmetric and in block form, with one of the blocks containing the dominant information of A, such as its largest eigenvalues. Moreover, the matrix M is constructed in a form that is easy to update when adding to A a symmetric rank-one matrix or when appending a row and, symmetrically, a column to A: the cost of such an updating is $O(n^2)$ floating point operations.

The proposed algorithm is based on the block anti-triangular form of the original matrix M, as introduced by the authors in [11]. Via successive orthogonal similarity transformations this form is then updated to a new form $A = \hat{Q}\hat{M}\hat{Q}^T$, whereby the first k rows and columns of $\hat{M}$ have elements bounded by a given threshold τ and the remaining bottom

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right part of $\hat{M}$ is maintained in block anti-triangular form. The updating transformations are all orthogonal, guaranteeing the backward stability of the algorithm, and the algorithm is very economical when the near rank deficiency is detected in some of the anti diagonal elements of the block anti-triangular form. Numerical results are also given showing the reliability of the proposed algorithm.

1. Introduction

Rank-revealing decompositions of dense matrices are widely used in applications such as signal and image processing, where accurate and reliable computation of the numerical rank, as well as the numerical range and null space, are required [4,12]. In such applications it is crucial to compute in a fast and reliable way an updating of such a factorization when a row or a column is appended/deleted to the initial matrix (updating/downdating) or when the initial matrix is modified by a symmetric rank-one matrix (rank-one modification).

For the unsymmetric case many rank-revealing algorithms have been proposed in the literature based on the QR factorization and URV decomposition [4,3,1,2,6]. The singular value decomposition (SVD) is of course a decomposition that reveals the numerical rank, but in general updatings or rank-one modifications cannot be computed in an efficient way [9,5].

In many applications the underlying matrix is symmetric [9,5] and it is therefore useful to consider rank revealing factorizations exploiting this symmetry. Recently, a new factorization of symmetric indefinite matrices $A = QMQ^T$, with Q orthogonal and M block-anti-triangular (BAT) has been introduced [11]. In particular, given a symmetric indefinite matrix $A \in \mathbb{R}^{n\times n}$ with inertia (n_-, n_0, n_+), the following decomposition can be computed,

$$A = QMQ^T, \quad M = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & Y^T \\ \mathbf{0} & \mathbf{0} & X & Z^T \\ \mathbf{0} & Y & Z & W \end{bmatrix} \begin{array}{l} \}n_0 \\ \}n_1 \\ \}n_2 \\ \}n_1 \end{array} \tag{1}$$

with $Q \in \mathbb{R}^{n\times n}$ orthogonal, $Z \in \mathbb{R}^{n_1\times n_2}$, $W \in \mathbb{R}^{n_1\times n_1}$ symmetric, $Y \in \mathbb{R}^{n_1\times n_1}$ nonsingular lower anti-triangular and $X \in \mathbb{R}^{n_2\times n_2}$ symmetric definite if $n_2 > 0$, i.e., $X = \omega LL^T$ with L lower triangular and

$$\omega = \begin{cases} 1, & \text{if } n_+ > n_- \\ -1, & \text{if } n_+ < n_- \end{cases}.$$