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Linearizations of matrix polynomials in Bernstein bases ☆

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ABSTRACT

We discuss matrix polynomials expressed in a Bernstein basis, and the associated polynomial eigenvalue problems. Using Möbius transformations of matrix polynomials, large new families of strong linearizations are generated. Matrix polynomials that are structured with respect to a Bernstein basis, together with their associated spectral symmetries, are also investigated. The results in this paper apply equally well to scalar polynomials, and include the development of new companion pencils for polynomials expressed in a Bernstein basis.

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1. Introduction

The now-classical scalar Bernstein polynomials were first used in [4] to provide a constructive proof of the Weierstrass approximation theorem, but since then have found numerous applications in computer-aided geometric design [5,16,19], interpolation and least squares problems [15,35,36], and statistical computing [37]. For additional applications of scalar Bernstein polynomials, as well as for more on the historical development and current research trends related to Bernstein polynomials, see [18,22] and the references therein.

This paper focuses on *matrix polynomials* expressed in Bernstein bases, and the associated polynomial eigenvalue problems. The classical approach for solving such eigenproblems is via a linearization, hence that notion takes a central role in this paper. When constructing linearizations for a matrix polynomial expressed in a Bernstein basis, $P(\lambda) = \sum_{i=0}^{k} A_i \beta_{i,k}(\lambda)$, it is important to avoid reformulating $P(\lambda)$ into the standard basis, since this change of basis can be poorly conditioned, and may introduce numerical errors [17,20]. Our main result is a simple procedure for systematically generating (strong) linearizations for such a $P(\lambda)$, using the matrix coefficients A_i directly. We show how this procedure can be used not only to recover the handful of previously known examples in the literature [1,26,40], but also to generate large new *families* of strong linearizations. Further, we study the impact that various matrix polynomial structures have on its spectrum, the existence of structured linearizations, and how existing structure-preserving algorithms can be applied. Along the way, we also describe a method for generating a family of new *companion pencils* for scalar polynomials expressed in a Bernstein basis.

It is important to emphasize that even though all the results in this paper are stated using the language of matrix polynomials, they also hold for the special case of *scalar* polynomials expressed in a Bernstein basis. Indeed, all applications involving polynomials in a Bernstein basis that are currently known to us are for the scalar case; thus the results in this paper are likely to first find application when computing with a Bernstein basis at the scalar level [5,40].

Following this introduction, some background on matrix polynomials and linearizations, as well as a brief review of the classical (scalar) Bernstein polynomials, is given in Section 2. Section 3 then introduces matrix polynomials expressed in Bernstein bases, and describes a simple method for generating strong linearizations of them. Finally, Section 4 describes some special spaces of linearizations for matrix polynomials in a Bernstein basis, while in Section 5 we discuss matrix polynomials that are structured with respect to a Bernstein basis.

2. Preliminaries

Throughout this paper $\mathbb{N}$ denotes the set of non-negative integers, $\mathbb{F}$ is an arbitrary field, $\overline{\mathbb{F}}$ denotes the algebraic closure of $\mathbb{F}$, and $\mathbb{F}_\infty := \mathbb{F} \cup \{\infty\}$. The ring of all univariate