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Polynomial reconstruction of signed graphs

Slobodan K. Simić [a,b,1], Zoran Stanić [c,*,2]

[a] State University of Novi Pazar, Vuka Karadžića bb, 36 300 Novi Pazar, Serbia
[b] Mathematical Institute SANU, Knez Mihailova 36, 11 000 Belgrade, Serbia
[c] Faculty of Mathematics, University of Belgrade, Studentski trg 16, 11 000 Belgrade, Serbia

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ABSTRACT

The reconstruction problem of the characteristic polynomial of graphs from their polynomial decks was posed in 1973. So far this problem is not resolved except for some particular cases. Moreover, no counterexample for graphs of order $n > 2$ is known. Here we put forward the analogous problem for signed graphs, and besides some general results, we resolve it within signed trees and unicyclic signed graphs, and also within disconnected signed graphs whose one component is either a signed tree or is unicyclic. A family of counterexamples that was encountered in this paper consists of two signed cycles of the same order, one balanced and the other unbalanced.

* Corresponding author.
E-mail addresses: sksimic@turing.mi.sanu.ac.rs (S.K. Simić), zstanic@matf.bg.ac.rs (Z. Stanić), zstanic@math.rs (Z. Stanić).

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1. Introduction

Given a simple graph $G = (V(G), E(G))$ of order $n = |V(G)|$ and size $m = |E(G)|$, let $\sigma : E(G) \mapsto \{+, -\}$ be a mapping defined on the edge set of G. Then, $\dot{G} = (G, \sigma)$ is a *signed graph* (or *sigraph*), G is its *underlying graph* and σ is its *sign function* (or *signature*).

The vertex set $V(\dot{G})$ of a signed graph $\dot{G}$ coincides with the vertex set of its underlying graph, while the edge set $E(\dot{G})$ is divided into two disjoint subsets E^+ and E^- (defined by σ) that contain positive and negative edges, respectively. If two vertices $u, v \in V(\dot{G})$ are joined by an edge, let $a_{uv} = \pm 1$ depending whether uv belongs to E^+ or E^-; otherwise, $a_{uv} = 0$. The adjacency matrix $A(\dot{G})$ of $\dot{G}$ is then, as expected, defined by $A(\dot{G}) = (a_{uv})$. Its characteristic polynomial

$$\Phi_{\dot{G}}(x) = \det(xI - A(\dot{G})) = x^n + a_{n-1}(\dot{G})x^{n-1} + \cdots + a_1(\dot{G})x + a_0(\dot{G}) \qquad (1)$$

is also called the *characteristic polynomial* of $\dot{G}$. The collection of eigenvalues of $\dot{G}$ (with repetition) is called the *spectrum* of $\dot{G}$. Note, since $A(\dot{G})$ is symmetric, the eigenvalues of $\dot{G}$ are real. We denote them by λ_i $(= \lambda_i(\dot{G}))$, where $i = 1, 2 \ldots, n$, and also assume that $\lambda_i \geq \lambda_j$ whenever $i < j$.

To avoid possible confusion, we make sure that signed graphs are recognized in the text by the attribute ‘signed’, while the graphs whose all edges are positive are referred to as ‘simple graphs’ or just ‘graphs’, and similarly for specific graphs like trees or cycles.

Most of the concepts defined for graphs are directly extended to signed graphs. For example, this refers to connectedness and bipartiteness, and also matchings (including perfect ones). If considering subgraphs of signed graphs, then their sign functions are the restrictions of the former ones to the corresponding edge subsets. Therefore, if H is a subgraph of G (not necessarily an induced one), then $\dot{H}$ stands for the resulting signed subgraph. If v is a vertex of G (or of $\dot{G}$) then we write $G - v$ (resp. $\dot{G} - v$) for the corresponding vertex-deleted subgraph. Most of the standard graph invariants coincide for G and $\dot{G}$. For example, $\delta(\dot{G}) = \delta(G) = \min\{\deg(v) : v \in V(G)\}$, where $\deg(v)$ is a degree of a vertex v (in G, or in $\dot{G}$). We also have that $g(\dot{G}) = g(G)$, where g is the *girth* of a graph, i.e. the length of its shortest (signed) cycle. Let

$$\mathcal{P}(\dot{G}) \; = \; \{\Phi_{\dot{G}_1}, \Phi_{\dot{G}_2}, \ldots, \Phi_{\dot{G}_n}\},$$

where $\dot{G}_i = \dot{G} - v_i$ $(1 \leq i \leq n)$, be the collection of characteristic polynomials of vertex-deleted subgraphs of $\dot{G}$. $\mathcal{P}(\dot{G})$ is also called the *polynomial deck* of $\dot{G}$. In this paper we consider the following problem.

Problem 1. Given two signed graphs $\dot{G}$ and $\dot{G}'$ on at least three vertices, is it true that

$$\mathcal{P}(\dot{G}) = \mathcal{P}(\dot{G}') \Longrightarrow \Phi_{\dot{G}} = \Phi_{\dot{G}'},$$