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On non-stationary cyclic convolution and combination

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ABSTRACT

We provide results on matrices arising in the non-stationary cyclic convolution and combination and its Fourier Transform. Spectral properties for these matrices, along with results on the non-stationary frequency filtering, are given.

1. The stationary case

Let $\mathbf{C}^n$ denote a n-dimensional complex vector space and consider $\mathbf{x} = (x_0, x_1, \ldots, x_{n-1})^T \in \mathbf{C}^n$. Let $G = \mathbf{Z}_n$ be the cyclic group of n elements. Consider a cyclic convolution operator by the mask $\mathbf{c} = (c_0, c_1, \ldots, c_{n-1})^T \in \mathbf{C}^n$ given by

$$y(n) = \sum_{k=0}^{n-1} c_k \mathbf{x}(n-k).$$

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In applications, the mask $\mathbf{c}$ is real valued and is used to filter/attenuate certain frequencies. This convolution operator can be represented with respect to the standard basis as the circulant matrix. For example, let $n = 4$ and set $\mathbf{c} = (c_0, c_1, c_2, c_3)^T$. Then the corresponding circulant matrix is given by

$$C = \begin{pmatrix} c_0 & c_3 & c_2 & c_1 \\ c_1 & c_0 & c_3 & c_2 \\ c_2 & c_1 & c_0 & c_3 \\ c_3 & c_2 & c_1 & c_0 \end{pmatrix}.$$

Denote $G = \{1, g, g^2, \cdots, g^{n-1}\}$ and let $\mathbf{C}[G]$ be the group algebra over $\mathbf{C}$ and let $\mathbf{M}$ be the group algebra all circulant matrices over $\mathbf{C}$. There is a natural group algebra isomorphism

$$\mathbf{C}[G] \to \mathbf{M}$$

given by

$$g \mapsto R$$

where R is the right shift circulant matrix

$$R_g = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Changing the basis to the Fourier domain, the complex exponentials

$$\xi_j = e^{\frac{2\pi i}{n} j(.)}$$

become the mask independent eigenvectors for the circulant matrix C. The corresponding eigenvalues $\{\lambda_j\}_{j=0}^{n-1}$ are given by

$$\lambda_j = \sum_{k=0}^{n-1} c_k e^{-\frac{2\pi i}{n} jk}$$

which is the Fourier transform of the mask $\mathbf{c}$. For any further reference on circulant matrices see [2], for example.

2. The non-stationary case

Non-stationary filtering is a common practice in seismic data processing. Many seismic processing techniques, that are not directly formulated as non-stationary filters, can