Linear Algebra and its Applications 499 (2016) 79–89

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

On energy of line graphs

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A R T I C L E I N F O

Article history:
Received 22 December 2015
Accepted 3 March 2016
Available online 14 March 2016
Submitted by R. Brualdi

MSC:
05C50
15A18

Keywords:
Graph spectrum
Signless Laplacian spectrum (of graph)
Energy (of graph)
Line graph

A B S T R A C T

The energy $\mathcal{E}(G)$ of a simple graph G is the sum of absolute values of the eigenvalues of its adjacency matrix. The line graph of G is denoted by L_G. We obtain a relation between $\mathcal{E}(L_G)$ and $\mathcal{E}(G)$, and establish bounds on $\mathcal{E}(L_G)$. Moreover, we present results pertaining to equienergetic and hyperenergetic graphs.

1. Introduction

In this paper we consider simple, undirected and unweighted graphs. Let $G = (V, E)$ be such a graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G)$, where

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http://dx.doi.org/10.1016/j.laa.2016.03.003

$|V(G)| = n$ and $|E(G)| = m$. For $v_i \in V(G)$, let d_i be the degree of the vertex v_i. The minimum vertex degree is denoted by $\delta = \delta(G)$. A vertex is said to be *pendent* if its neighborhood contains exactly one vertex. An edge is said to be *pendent* if one of its end vertices is pendent. An *isolated vertex* is a vertex with degree zero, that is, a vertex that is not an endpoint of any edge. Let p and s be, respectively, the number of pendent and isolated vertices of the graph G. In addition, let $\mathbf{A}(G)$ be the *adjacency matrix* of G and $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{n-1} \geq \lambda_n$ denote the eigenvalues of $\mathbf{A}(G)$. Sometimes, by convenience, we write $\lambda_i = \lambda_i(G)$.

The *energy* of the graph G is defined as

$$\mathcal{E}(G) = \sum_{i=1}^{n} |\lambda_i|.$$

Its mathematical properties were extensively investigated, see the book [29], the recent articles [13,19,27,28,30,36] and the references cited therein. The *positive (resp. negative) inertia* of the graph G, denoted by ν^+ (resp. ν^-), is the number of the positive (resp. negative) eigenvalues of $\mathbf{A}(G)$. The *rank of a graph G* is the rank of its adjacency matrix $\mathbf{A}(G)$, equal to $\nu^+ + \nu^-$. Recall that $\nu^+ \geq 1$ holds if and only if the underlying graph has at least one edge.

Let $\mathbf{L}(G) = \mathbf{D}(G) - \mathbf{A}(G)$ and $\mathbf{Q}(G) = \mathbf{D}(G) + \mathbf{A}(G)$ be, respectively, the *Laplacian matrix* [20,21,31] and the *signless Laplacian matrix* [5,7,8] of the graph G, where $\mathbf{D}(G)$ is the diagonal matrix of vertex degrees. The eigenvalues of $\mathbf{L}(G)$ and $\mathbf{Q}(G)$ will be denoted by $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_{n-1} \geq \mu_n = 0$ and $q_1 \geq q_2 \geq \cdots \geq q_{n-1} \geq q_n$, respectively. Then the *Laplacian energy* and the *signless Laplacian energy* of G are defined as

$$LE = LE(G) = \sum_{i=1}^{n} \left| \mu_i - \frac{2m}{n} \right| \quad \text{and} \quad LE^+ = LE^+(G) = \sum_{i=1}^{n} \left| q_i - \frac{2m}{n} \right|$$

respectively. The Laplacian energy is nowadays reasonably well elaborated (see [10,11,24,33] and the references cited therein). The signless Laplacian energy was until now studied only to a limited extent [1,11]. Moreover, $LE^+(G) = LE(G)$ for bipartite graphs G.

The *line graph* L_G of the graph G is the graph whose vertex set is in one-to-one correspondence with the set of edges of G, where two vertices of L_G are adjacent if and only if the corresponding edges in G have a vertex in common [25]. The energy of line graph and its relations with other graph energies were earlier studied in [12,23].

As usual, K_n, C_n, P_n, and $K_{1,n-1}$, denote, respectively, the complete graph, cycle, path and star on n vertices. For other undefined notations and terminology from graph theory, the readers are referred to [3].

The paper is organized as follows. In Section 2, we give a list of some previously known results. In Section 3, we obtain a relation between $\mathcal{E}(L_G)$ and $\mathcal{E}(G)$. In Section 4, we establish bounds on $\mathcal{E}(L_G)$ in terms of n, m, p, and s. Section 5 outlines results pertaining to equienergetic and hyperenergetic graphs.