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Relationships between Brauer-type eigenvalue inclusion sets and a Brualdi-type eigenvalue inclusion set for tensors

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The comparisons of the Brualdi-type eigenvalue inclusion set provided by Bu et al. (2015) [3] and the Brauer-type eigenvalue inclusion set provided by Li and Li (2015) [6], are established. In particular, a condition such that the Brualdi-type eigenvalue inclusion set is tighter than the Brauer-type eigenvalue inclusion set, is given.

1. Introduction

Let $\mathbb{C}$ ($\mathbb{R}$) denote the set of all complex (real) numbers and $N = \{1, 2, \ldots, n\}$. We call $\mathcal{A} = (a_{i_1 \cdots i_m})$ a complex (real) tensor of order m dimension n, denoted by $\mathcal{A} \in \mathbb{C}^{[m,n]}$ ($\mathcal{A} \in \mathbb{R}^{[m,n]}$, resp.), if

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$$a_{i_1\cdots i_m} \in \mathbb{C}\ (\mathbb{R}),$$

where $i_j = 1, \ldots, n$ for $j = 1, \ldots, m$. A real tensor $\mathcal{A} = (a_{i_1\cdots i_m})$ is called symmetric [7,8,11] if

$$a_{i_1\cdots i_m} = a_{\pi(i_1\cdots i_m)}, \forall \pi \in \Pi_m,$$

where Π_m is the permutation group of m indices. Furthermore, a real tensor of order m dimension n is called the unit tensor, if its entries are $\delta_{i_1\cdots i_m}$ for $i_1, \ldots, i_m \in N$, where

$$\delta_{i_1\cdots i_m} = \begin{cases} 1, & \text{if } i_1 = \cdots = i_m, \\ 0, & \text{otherwise.} \end{cases}$$

For a tensor $\mathcal{A} = (a_{i_1\cdots i_m}) \in \mathbb{C}^{[m,n]}$, and a vector $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{C}^n$, $\mathcal{A}x^{m-1}$ is defined as an n dimension vector whose ith component is

$$(\mathcal{A}x^{m-1})_i = \sum_{i_2,\ldots,i_m \in N} a_{ii_2\cdots i_m} x_{i_2} \cdots x_{i_m}.$$

Moreover, if a complex number λ and a nonzero complex vector $x = (x_1, x_2, \ldots, x_n)^T$ satisfy

$$\mathcal{A}x^{m-1} = \lambda x^{[m-1]},$$

then λ is called an eigenvalue of $\mathcal{A}$ and x an eigenvector of $\mathcal{A}$ associated with λ, where

$$x^{[m-1]} = (x_1^{m-1}, x_2^{m-1}, \ldots, x_n^{m-1})^T.$$

This definition was introduced by Qi in [8] where he assumed that $\mathcal{A} \in \mathbb{R}^{[m,n]}$ is symmetric and m is even. Independently, in [7], Lim gave such a definition but restricted x to be a real vector and λ to be a real number.

There has been extensive attention and interest in spectral theory of tensors [3,5–7,11] and hypergraphs [9]. Qi [8] extend the well-known Geršgorin eigenvalue inclusion set [4,10] of matrices to real symmetric tensors of higher order. And this result can be easily generalized to general tensors [5,11].

Theorem 1. *(See [5,8,11].) Let $\mathcal{A} = (a_{i_1\cdots i_m}) \in \mathbb{C}^{[m,n]}$. Then*

$$\sigma(\mathcal{A}) \subseteq G(\mathcal{A}) = \bigcup_{i \in N} G_i(\mathcal{A}),$$

where $\sigma(\mathcal{A})$ is the set of all the eigenvalues of $\mathcal{A}$ and

$$G_i(\mathcal{A}) = \{z \in \mathbb{C} : |z - a_{i\cdots i}| \le r_i(\mathcal{A})\},\ r_i(\mathcal{A}) = \sum_{\substack{i_2,\ldots,i_m \in N, \\ \delta_{ii_2\ldots i_m}=0}} |a_{ii_2\cdots i_m}|.$$