

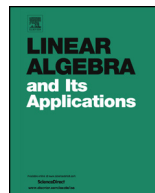


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On the diagonalizability of a matrix by a symplectic equivalence, similarity or congruence transformation

Ralph John de la Cruz^{a,*}, Heike Faßbender^b

^a *Institute of Mathematics, University of the Philippines, Diliman, Quezon City 1101, Philippines*

^b *AG Numerik, Institut Computational Mathematics, TU Braunschweig, D-38092 Braunschweig, Germany*

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ABSTRACT

A symplectic matrix $S \in \mathbb{C}^{2n \times 2n}$ satisfies $S = J^{-1}S^T J$ for $J = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \in \mathbb{R}^{2n \times 2n}$. We will consider symplectic equivalence, similarity and congruence transformations and answer the question under which conditions a $2n \times 2n$ matrix is diagonalizable under one of these transformations. In particular, we will give symplectic analogues of the singular value decomposition and the Takagi factorization.

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1. Introduction

It is well known that

* Corresponding author.

E-mail addresses: rjdelacruz@math.upd.edu.ph (R.J. de la Cruz), h.fassbender@tu-braunschweig.de (H. Faßbender).

- Any matrix $A \in \mathbb{C}^{n \times n}$ is unitarily equivalent to a diagonal matrix (singular value decomposition (SVD) [6, Section 3.4]).
- A matrix $A \in \mathbb{C}^{n \times n}$ is unitarily similar to a diagonal matrix if and only if A is normal (that is, $AA^* = A^*A$ where A^* denotes the conjugate transpose of A [6, Theorem 2.5.4]).
- A matrix $A \in \mathbb{C}^{n \times n}$ is unitarily congruent to a diagonal matrix if and only if A is symmetric (Takagi factorization [6, Corollary 4.4.4]).

Note that unitary matrices are those matrices which preserve length in the Euclidean norm. That is, those matrices $U \in \mathbb{C}^{n \times n}$ satisfying $[Ux, Uy] = [x, y]$ for all $x, y \in \mathbb{C}^n$ where $[\cdot, \cdot]$ denotes the Euclidean inner product on $\mathbb{C}^n$

$$[x, y] = x^*y.$$

Observe that U is unitary if and only if $UU^* = U^*U = I$ and that a unitary matrix is necessarily normal. Clearly, the adjoint of a matrix $A \in \mathbb{C}^{n \times n}$ with respect to $[\cdot, \cdot]$ is given by A^* .

Here we will discuss symplectic analogues of the above statements. The symplectic matrices are the isometries of the bilinear form $\langle \cdot, \cdot \rangle_{J_{2n}}$ associated with $J_{2n} \equiv \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}$

$$\langle x, y \rangle_{J_{2n}} \equiv x^T J_{2n} y$$

for all $x, y \in \mathbb{C}^{2n}$ (that is, any matrix P with $\langle Px, Py \rangle_{J_{2n}} = \langle x, y \rangle_{J_{2n}}$, for all $x, y \in \mathbb{C}^{2n}$ is symplectic). Observe that $A \in \mathbb{C}^{2n \times 2n}$ is symplectic if and only if $A^T J_{2n} A = J_{2n}$. Clearly, the adjoint of a matrix $A \in \mathbb{C}^{2n \times 2n}$ with respect to $\langle \cdot, \cdot \rangle_{J_{2n}}$ is given by $J_{2n}^{-1} A^T J_{2n}$. We omit the subscript if the size of J is clear from the context. For the ease of notation, let us define the operator $\phi_J : \mathbb{C}^{2n \times 2n} \rightarrow \mathbb{C}^{2n \times 2n}$ by

$$\phi_J(A) = J^{-1} A^T J. \quad (1)$$

In analogy to the definition of normal matrices, a matrix $A \in \mathbb{C}^{2n \times 2n}$ will be called ϕ_J normal if

$$\phi_J(A)A = A\phi_J(A). \quad (2)$$

In particular, we will prove

- A matrix $A \in \mathbb{C}^{2n \times 2n}$ is symplectically equivalent to a diagonal matrix if and only if $A\phi_J(A)$ is similar to $\phi_J(A)A$ and $A\phi_J(A)$ is diagonalizable (symplectic SVD).
- A matrix $A \in \mathbb{C}^{2n \times 2n}$ is symplectically similar to a diagonal matrix if and only if $\phi_J(A)A = A\phi_J(A)$ (that is, A is ϕ_J normal) and A is diagonalizable.
- A matrix $A \in \mathbb{C}^{2n \times 2n}$ is symplectically congruent to a diagonal matrix if and only if A is symmetric and $A\phi_J(A)$ is diagonalizable (symplectic Takagi factorization).

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