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A novel representation of rank constraints for real matrices ☆

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ABSTRACT

We present a novel representation of rank constraints for non-square real matrices. We establish relationships with existing results and show that these are particular cases of our representation. One of these cases is a representation of the ℓ_0 pseudo-norm, which is used in sparse representation problems. Finally, we describe how our representation can be included in rank-constrained optimization and in rank-minimization problems.

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1. Introduction

Rank constraints find application in many areas including data modelling, systems and control, computer algebra, signal processing, psychometrics, machine learning, computer vision, among others [1,2]. In many applications, the notion of complexity of a model can be related to the rank of a particular matrix. For example, in factor analysis, the number of latent factors is equal to the rank of a covariance matrix. In system identification, the order of a rational system is equal to the rank of an infinite dimensional Hankel matrix.

Handling rank constraints is known to be difficult since the rank$(\cdot)$ function has undesirable features. In particular, the function is non-smooth, non-linear and non-convex. In applications based on optimization, where smoothness and convexity are widely exploited, the non-smoothness of the rank$(\cdot)$ function limits the tools that can be used in the optimization problem. On the other hand, non-smoothness is often tolerated in order to obtain other desirable properties, e.g. convexity, in some other part of the problem. This has motivated several authors [3–5,1,6] to find equivalent representations for rank constraints. These representations are equivalent ways to express a rank constraint of the form $\text{rank}(A) \leq r$. These equivalent representations are aimed at overcoming the non-linearity, non-smoothness and/or non-convexity of the rank function. For example, one equivalent representation of a rank constraint for $A \in \mathbb{R}^{m\times n}$ is given by

$$\text{rank}(A) \leq r \iff \exists \text{ a full row rank matrix } U \in \mathbb{R}^{(m-r)\times m} \text{ such that } UA = 0 \quad (1)$$

One advantage of the rank representation (1) is that it frees the matrix A to satisfy other structural constraint. In [1] the rank representation (1) has been used to impose a rank constraint in a structured matrix, such as a Hankel matrix. However, this rank constraint representation have some limitations. First, it transfers a rank constraint from a matrix A, to an auxiliary matrix U. Moreover, the rank of the matrix A is related to the size of the auxiliary matrix U. This last issue makes this approach inappropriate for problems where the rank to be constrained is selected in a dynamic way, e.g. online.

Other existing rank constraint representations are valid only for positive semidefinite matrices, see e.g. [3,5,4]. For example, consider the rank constraint representation in [3] that establishes that, for a matrix $A \in \mathbb{S}^n_+$, then

$$\text{rank}(A) \leq r \iff \exists W \in \Phi_{n,r} \text{ such that } \text{trace}(AW) = 0 \quad (2)$$

where

$$\Phi_{n,r} = \{W \in \mathbb{S}^n,\ 0 \preceq W \preceq I, \text{trace}(W) = n - r\} \quad (3)$$

This rank constraint representation eliminates of the rank function, but is only valid for positive semidefinite matrices. A detailed discussion of these and other rank representations is given in section 2.

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