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On the structure of graded Leibniz triple systems ☆

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ABSTRACT

We study the structure of a Leibniz triple system $\mathcal{E}$ graded by an arbitrary abelian group G which is considered of arbitrary dimension and over an arbitrary base field $\mathbb{K}$. We show that $\mathcal{E}$ is of the form $\mathcal{E} = U + \sum_{[j]\in\sum^1/\sim} I_{[j]}$ with U a linear subspace of the 1-homogeneous component $\mathcal{E}_1$ and any ideal $I_{[j]}$ of $\mathcal{E}$, satisfying $\{I_{[j]}, \mathcal{E}, I_{[k]}\} = \{I_{[j]}, I_{[k]}, \mathcal{E}\} = \{\mathcal{E}, I_{[j]}, I_{[k]}\} = 0$ if $[j] \neq [k]$, where the relation $\sim$ in $\sum^1 = \{g \in G\backslash\{1\} : L_g \neq 0\}$, defined by $g \sim h$ if and only if g is connected to h.
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1. Introduction

Leibniz triple systems were introduced by Bremner and Sánchez-Ortega [1]. Leibniz triple systems were defined in a functorial manner using the Kolesnikov–Pozhidaev algorithm, which took the defining identities for a variety of algebras and produced the

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defining identities for the corresponding variety of dialgebras [2]. In [1], Leibniz triple systems were obtained by applying the Kolesnikov–Pozhidaev algorithm to Lie triple systems. The study of gradings on Lie algebras begins in the 1933 seminal Jordan's work, with the purpose of formalizing Quantum Mechanics [3]. Since then, the interest on gradings on different classes of algebras has been remarkable in the recent years, motivated in part by their application in physics and geometry [4–7]. Recently, in [7–12], the structure of arbitrary graded Lie algebras, graded Lie superalgebras, graded commutative algebras, graded Leibniz algebras and graded Lie triple systems have been determined by the techniques of connections of roots. Our work is essentially motivated by the work on graded Lie triple systems [12] and the work on split Leibniz triple systems [13].

Throughout this paper, Leibniz triple systems $\mathcal{E}$ are considered of arbitrary dimension and over an arbitrary base field $\mathbb{K}$. This paper proceeds as follows. In section 2, we establish the preliminaries on graded Leibniz triple systems theory. In section 3, we show that such an arbitrary Leibniz triple system is of the form $\mathcal{E} = U + \sum_{[j]\in\sum^1/\sim} I_{[j]}$ with U a subspace of $\mathcal{E}_1$ and any ideal $I_{[j]}$ of $\mathcal{E}$, satisfying $\{I_{[j]}, \mathcal{E}, I_{[k]}\} = \{I_{[j]}, I_{[k]}, \mathcal{E}\} = \{\mathcal{E}, I_{[j]}, I_{[k]}\} = 0$ if $[j] \neq [k]$, where the relation $\sim$ in $\sum^1$, defined by $g \sim h$ if and only if g is connected to h.

2. Preliminaries

Definition 2.1. (See [14].) A **right Leibniz algebra** L is a vector space over a base field $\mathbb{K}$ endowed with a bilinear product $[\cdot,\cdot]$ satisfying the Leibniz identity

$$[[y, z], x] = [[y, x], z] + [y, [z, x]],$$

for all $x, y, z \in L$.

Definition 2.2. (See [1].) A **Leibniz triple system** is a vector space $\mathcal{E}$ endowed with a trilinear operation $\{\cdot,\cdot,\cdot\} : \mathcal{E} \times \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ satisfying

$$\{a, \{b, c, d\}, e\} = \{\{a, b, c\}, d, e\} - \{\{a, c, b\}, d, e\} - \{\{a, d, b\}, c, e\} + \{\{a, d, c\}, b, e\}, \tag{2.1}$$

$$\{a, b, \{c, d, e\}\} = \{\{a, b, c\}, d, e\} - \{\{a, b, d\}, c, e\} - \{\{a, b, e\}, c, d\} + \{\{a, b, e\}, d, c\}, \tag{2.2}$$

for all $a, b, c, d, e \in \mathcal{E}$.

Example 2.3. A Lie triple system gives a Leibniz triple system with the same ternary product. If L is a Leibniz algebra with product $[\cdot,\cdot]$, then L becomes a Leibniz triple system by putting $\{x, y, z\} = [[x, y], z]$. More examples refer to [1].

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