Linear Algebra and its Applications 495 (2016) 38–50

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

A non-modulus linear method for solving the linear complementarity problem

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ARTICLE INFO

Article history:
Received 8 December 2015
Accepted 18 January 2016
Available online 22 January 2016
Submitted by R. Brualdi

MSC:
65F08
65F10

Keywords:
Linear complementarity problem
Modulus-based method
Sign pattern

ABSTRACT

In this paper, a non-modulus linear method for solving the linear complementarity problem is established by using the sign patterns of the solution of the equivalent modulus equation. In the proposed method the efficient numerical algorithms for solving the linear equations can be applied to the large sparse problems. Numerical examples show that the new method is valid.

1. Introduction

The linear complementarity problem, abbreviated as LCP(q, A), consists of finding vectors $z \in \mathbf{R}^n$ such that

$$r = Az + q \geq 0,\ z \geq 0, \quad \text{and} \quad z^T r = 0, \tag{1}$$

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http://dx.doi.org/10.1016/j.laa.2016.01.032

where $A \in \mathbf{R}^{n\times n}$ and for two $m \times n$ matrices $B = (b_{ij})$ and $C = (c_{ij})$ the order $B \geq C$ means $b_{ij} \geq c_{ij}$ for any i and j.

The linear complementarity problem has many applications, e.g., in the free boundary problems, the network equilibrium problems and the contact problems, etc. (e.g., see [13, 14] and the references therein).

It is known from [12] that LCP(q, A) is completely equivalent to solving the next equation

$$(A + I)x + (A - I)|x| + q = 0, \tag{2}$$

and x is the solution of (2) only if $z = x + |x|$ is the solution of (1). Recently, many articles gave some solvers of LCP(q, A) based on (2). In particular, Bai presented a modulus-based matrix splitting method for solving LCP(q, A), and presented convergence analysis for the proposed methods; see [4]. In [18] the author gave a framework for the modulus-based matrix splitting iteration method. For more works about the modulus-based iteration method see [2,3,6–10,19,27–33]. The existing efficient technique to solve (2) is based on matrix splittings. However, how to choose the positive diagonal parameter matrices for fast convergence in the modulus-based methods is unknown.

It is well known that if the sign patterns of the solution of (2) is known, the LCP(q, A) can be solved efficiently. In [17], Kakimura introduced the sign-solvable LCPs with nonzeros diagonals and proposed a polynomial time algorithm to solve them from the sign patterns of A and q. However, the assumption in [17], that the LCP(q, A) is totally sign-nonsingular, is difficult to satisfied. In this paper we propose a non-modulus linear method to solve (1) by using the sign pattern of the solution of (2) under some other simpler assumptions.

In order to give our method, first we introduce the notation and definitions.

Let $A = (a_{ij}) \in \mathbf{R}^{n\times n}$. By $|A|$ we denote $|A| = (|a_{ij}|)$. A matrix A is called (e.g., see [11]) a Z-matrix, if $a_{ij} \leq 0$ for any $i \neq j$; a P-matrix, if $x_i(Ax)_i > 0$ for all $x \neq 0$; an M-matrix, if $A^{-1} \geq 0$ and A is a Z-matrix; a positive definite matrix, if $x^T Ax > 0$ for any $x \neq 0$. It is well known that LCP (1) has a unique solution for any vector $q \in \mathbf{R}^n$ if and only if $A = (a_{ij})$ is a P-matrix [13]. Let $e = (1, 1, \cdots, 1)^T \in \mathbf{R}^n$.

Let $\langle n\rangle = \{1, 2, \cdots, n\}$. The index sets of positive elements, negative elements and zero elements of x are denoted by

$$\mathbb{P}_x = \{j | j \in \langle n\rangle, x_j > 0\},$$
$$\mathbb{N}_x = \{j | j \in \langle n\rangle, x_j < 0\}$$

and

$$\mathbb{Z}_x = \{j | j \in \langle n\rangle, x_j = 0\},$$