Linear Algebra Appl. 495 (2016) 163–173

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Bounds for the 1-norm of the inverses of some triangular matrices

Higidio Portillo Oquendo [a,*], Patricia Sánez Pacheco [b]

[a] Department of Mathematics, Federal University of Paraná, Brazil
[b] Department of Mathematics, Federal Technological University of Paraná, Brazil

ARTICLE INFO

Article history:
Received 23 July 2014
Accepted 14 January 2016
Available online 29 January 2016
Submitted by V. Mehrmann

MSC:
15A09
39A10
34K28
47B35
15A57

Keywords:
Triangular matrix
Uniform bounds
Inverse matrix
Toeplitz matrix

ABSTRACT

In this paper, we find bounds for the 1-norm of the inverses of triangular matrices whose columns may decrease to zero. These results complement those obtained by Berenhaut et al. in [1] where the columns cannot decrease to zero. Also, we extend the 1-norm estimates obtained for Toeplitz matrices, by Liu et al. in [5], to a non-Toeplitz class. Finally, we construct a class of matrices where the estimates obtained are the best.

1. Introduction

Uniform estimates for the norms of the inverses of matrices play an important role in the computational resolution of systems of equations arising from discretizations of some

* Corresponding author.
E-mail addresses: higidio@ufpr.br (H. Portillo Oquendo), patricias@utfpr.edu.br (P. Sánez Pacheco).

http://dx.doi.org/10.1016/j.laa.2016.01.028

problems in applied sciences. In this sense, we are interested to find suitable estimates for 1-norms of the inverses of triangular matrices of the form

$$B_n = \begin{pmatrix} b_{11} & & & \\ b_{21} & b_{22} & & \\ \vdots & & \ddots & \\ b_{n1} & & b_{n,n-1} & b_{nn} \end{pmatrix},$$

where the coefficients (b_{ij}), $i \geq j \geq 1$ are positive. It is known that its inverse matrix is of the same type, namely

$$B_n^{-1} = \begin{pmatrix} x_{11} & & & \\ x_{21} & x_{22} & & \\ \vdots & & \ddots & \\ x_{n1} & & x_{n,n-1} & x_{nn} \end{pmatrix}.$$

In this case, the coefficients of B_n and B_n^{-1} are related as follows: $b_{jj}x_{jj} = 1$ and $\sum_{k=j}^{i} b_{ik}x_{kj} = 0$ for all $i > j \geq 1$. Therefore, for each fixed $j \geq 1$, x_{ij} is the solution of the following initial value problem: for $i > j$,

$$x_{ij} = -\sum_{k=j}^{i-1} \frac{b_{ik}}{b_{ii}} x_{kj} \quad \text{with initial data} \quad x_{jj} = 1/b_{jj}.$$

Uniform estimates for the 1-norm of inverses of matrices defined by

$$\|B_n^{-1}\|_1 = \max_{1\leq j\leq n} \sum_{i=j}^{n} |x_{ij}|,$$

were obtained by several authors, some of them can be found in [1–3,5–7]. In particular, we shall mention some of them: in [1], Berenhaut et al. found uniform bounds for the 1-norm of a class of inverses of matrices, obtaining the following result

Theorem 1.1. *(See [1,2].) Assume that*

1. $b_{ij} \geq \beta_0 > 0$, *for* $i \geq j \geq 1$,
2. $b_{i+1,j} \leq b_{ij}$, *for* $i \geq j \geq 1$.

Then, we have the uniform bound:

$$\|B_n^{-1}\|_1 \leq \frac{2}{\beta_0}, \quad \forall n \geq 1.$$