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Ordering of some uniform supertrees with larger spectral radii ☆

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A B S T R A C T

A supertree is a connected and acyclic hypergraph. We study some uniform supertrees with larger spectral radii. We first define a new type of edge-moving operation on hypergraphs, and study its applications on the comparison of the spectral radii of hypergraphs. By using this new operation on some supertrees together with the general edge-moving operation introduced by Li, Shao and Qi in 2015, and using a result by Zhou et al. in 2014 about the relation between the spectral radii of an ordinary graph and its power hypergraph, we are able to determine the first eight k-uniform supertrees on n vertices with the largest spectral radii.

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1. Introduction

In recent years, the study of the spectra of tensors and hypergraphs with their various applications has attracted extensive attention and interest, since the work of Qi [12] and Lim [10].

In 2005, Qi [12] and Lim [10] independently introduced the concept of tensor eigenvalues and the spectra of tensors. In 2008, Lim [11] proposed the study of the spectra of hypergraphs via the spectra of tensors.

In 2012, Cooper and Dutle [3] defined the eigenvalues (and the spectrum) of a uniform hypergraph as the eigenvalues (and the spectrum) of the adjacency tensor of that hypergraph, and obtained a number of interesting results on the spectra of hypergraphs. They also generalized some basic spectral results from graphs to hypergraphs. The (adjacency) spectrum of uniform hypergraphs were further studied in [9,4,8,17,15].

In 2015 [9], Li, Shao and Qi studied some extremal spectral properties of the classes of k-uniform supertrees and hypertrees of order n, and determined that the hyperstar (which will be denoted by $S_{n'}^k$ later) attains uniquely the maximal spectral radius among all k-uniform supertrees of order n. They also determined the unique k-uniform supertree of order n with the second largest spectral radius.

In this paper, we first study a new type of edge-moving operation on hypergraphs (see Definition 3.1). By using a combination of this new operation together with the general edge-moving operation defined and studied in [9], we are able to determine the first eight k-uniform supertrees of order n with the largest spectral radii.

Definition 1.1. (1). Let a, b be positive integers with $a + b = n - 2$. Let $S(a, b)$ be the tree of order n obtained from an edge e by attaching a pendent edges to one end vertex of e, and attaching b pendent edges to the other end vertex of e. (Obviously we have $S(a, b) = S(b, a)$.)

(2). Let a, b, c be integers satisfying $a, c \geq 1$, $b \geq 0$ and $a+b+c = n-3$. Let $S(a, b, c)$ be the tree of order n obtained from a path P_3 by attaching b pendent edges to the center of P_3, and attaching a and c pendent edges to the two end vertices of P_3, respectively. (Obviously we have $S(a, b, c) = S(c, b, a)$.)

Let S_n be the star of order n. For convenience, we denote

$$\begin{aligned} &T_{n,1} = S_n, \quad T_{n,2} = S(1, n-3), \quad T_{n,3} = S(2, n-4), \quad T_{n,4} = S(1, n-5, 1), \\ &T_{n,5} = S(n-4, 0, 1), \quad T_{n,6} = S(3, n-5), \quad T_{n,7} = S(1, n-6, 2). \end{aligned} \tag{1.1}$$

The largest eigenvalue of the adjacency matrix of graph G is called the spectral radius of G, denoted by $\rho(G)$. It was proved in [7,2] that these 7 trees in (1.1) are the first 7 trees of order n with the largest spectral radii. Namely we have:

Theorem 1.1. *(See [7,2].) Let $n \geq 11$. Then we have the following ordering of the spectral radii of the trees of order n:*