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Sampling method based projection approach for the reconstruction of 3D acoustically penetrable scatterers

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We present a projection based regularization parameter choice approach within the framework of the linear sampling method for the reconstruction of acoustically penetrable objects. Using the Golub–Kahan bidiagonalization algorithm and the Lanczos tridiagonalization process we form appropriate subspaces which generate a sequence of regularized solutions. As a result two new and efficient methods are developed and used for the solution of problems that involve large linear systems of equations. The effectiveness of our approach is illustrated with reconstructions of three dimensional objects.

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Far-field matrix
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1. Introduction

In this work, we will deal with reconstructions of acoustically penetrable objects from far-field measurements. This problem was originally investigated by Anagnostopoulos et al. [1] within the framework of the factorization method. Our goal will be to construct a robust and effective algorithm that will be able to handle acoustically penetrable objects that exhibit large linear systems of equations via the linear sampling method originally developed by Colton and Kirsch [11].

It is widely known that the linear sampling method does not require *a priori* information about either the boundary condition or the connectivity of the scatterer, however requires knowledge of the far-field pattern for all incident and observation directions. Hence, the cost of reconstructing three dimensional objects via the linear sampling method (LSM) could be prohibitively expensive if the discretization involves the construction of large systems of equations. In addition, due to the ill-posedness of the inverse problem, the linear sampling method yields an ill-conditioned system of linear equations whose solution requires a regularization method in order to handle correctly the presence of noise in the data. Moreover, the noise level in the data should be known *a priori*, something that in real life applications is not the case in general.

In our approach we will use an appropriate projection method that through the LSM will construct stable approximations to the far-field equation. Due to noise in the data however, it is necessary to combine our projection method with a regularization method like Tikhonov's regularization equipped with Morozov's generalized discrepancy principle as parameter choice rule which generally involves the computation of the zeros of the discrepancy function at each point of the grid. For large systems however, SVD-based methods, like the latter may be prohibitive due to the huge amount of data.

In the sequel, we will introduce two methods whose main idea will be to approximate the regularized solution by using a sequence of regularized solutions in appropriate Krylov subspaces of increasing dimension generated by projection methods such as Golub–Kahan bidiagonalization (GKB) [18, Section 8.6.2] and Lanczos tridiagonalization methods [18, Chapter 9]. Therefore, we will be taking advantage of the fact that the regularization parameter for the projected problem, which involves a small number of variables, can be determined efficiently and at low computational cost using a direct method such as the SVD of a small matrix. The first parameter estimation method will be called PGDP-FP (Projected Generalized Discrepancy Principle Fixed Point method) and will enable the widely used Morozov's discrepancy principle method to effectively provide regularization parameters, for problems that involve large linear systems of equations, as roots of a projected discrepancy function. Taking into account that GDP can fail when the noise level is not accurately estimated, we developed a second method that