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Collocation–quadrature methods and fast summation for Cauchy singular integral equations with fixed singularities

P. Junghanns *, R. Kaiser, D. Potts

Chemnitz University of Technology, Faculty of Mathematics, Reichenhainer Str. 39, D-09107 Chemnitz, Germany

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ABSTRACT

Basing on recent results on the stability of collocation methods applied to Cauchy singular integral equations with additional fixed singularities we give necessary and sufficient conditions for the stability of collocation–quadrature methods for such equations. These methods have the advantage that the respective system of equations has a very simple structure and allows to apply fast summation methods which results in a fast algorithm with $\mathcal{O}(n \log n)$ complexity. We present numerical results of the application of the proposed collocation–quadrature methods to the notched half plane problem of two-dimensional elasticity theory.

1. Introduction

In the paper [1] the stability of collocation methods applied to Cauchy singular integral equations with additional Mellin-type operators (cf. (2.1)) was studied. The present

* Corresponding author.
E-mail addresses: peter.junghanns@mathematik.tu-chemnitz.de (P. Junghanns), robert.kaiser@mathematik.tu-chemnitz.de (R. Kaiser), daniel.potts@mathematik.tu-chemnitz.de (D. Potts).

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paper is a continuation of these investigations focusing on a further discretization step applied to the integral operators with fixed singularities by applying an appropriate quadrature rule. These collocation–quadrature methods, which we introduce here, have the advantage (in comparison to the pure collocation methods) that the respective system of discrete equations has a very simple structure, which enables us to apply fast summation methods resulting in a fast algorithm with $\mathcal{O}(n \log n)$ complexity. Let us remark that the methods under consideration here are based on the approximation of the unknown solution by weighted polynomials and collocating at the zeros of Chebyshev polynomials. Concerning collocation and quadrature methods, which are based on spline approximation using suitably graded meshes, we refer to Chapter 11 of [2] or [3] and the literature cited there. Here we will see that, in many situations, the set of zeros of orthogonal polynomials as the set of collocation points is already suitably graded.

The present paper is organized as follows. In Section 2 the collocation–quadrature methods, we deal with here, are introduced, Section 3 contains a short description of the C^*-algebra background, which we use for proving stability of the methods, and Section 4 shows that the operator sequences of the collocation–quadrature methods belong to the C^*-algebra defined in Section 3. In Section 5 we prove the main result, namely the stability theorem, and in Section 6 we describe the structure of the systems of discrete equations and the application of a fast summation method. In Section 7 we present numerical results obtained by applying the investigated methods to the notched half plane problem of two-dimensional elasticity theory, and the final Section 8 is devoted to the proof of convergence rates for the collocation and the collocation–quadrature methods.

2. The quadrature method

Here we consider the Cauchy singular integral equation with additional fixed singularities

$$\begin{aligned} a(x)u(x) + \frac{b(x)}{\pi \mathbf{i}} \int_{-1}^{1} \frac{u(y)\,dy}{y-x} + \sum_{k=1}^{m_-} \beta_k^- \int_{-1}^{1} \mathbf{h}_k^- \left(\frac{1+x}{1+y} \right) \frac{u(y)\,dy}{1+y} \\ + \sum_{k=1}^{m_+} \beta_k^+ \int_{-1}^{1} \mathbf{h}_k^+ \left(\frac{1-x}{1-y} \right) \frac{u(y)\,dy}{1-y} = f(x), \end{aligned} \tag{2.1}$$

$-1 < x < 1$, where $\beta_k^\pm \in \mathbb{C}$ and $m_\pm \in \mathbb{N}$ are given numbers and where the functions $\mathbf{h}_k^\pm$ are defined by

$$\mathbf{h}_k^\pm(s) = \frac{(\mp 1)^k}{\pi \mathbf{i}} \frac{s^{k-1}}{(1+s)^k}\,, \quad s > 0\,,\ k \in \mathbb{N}\,.$$