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Some inverse limit approaches to the Riordan group

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ABSTRACT

Using the inverse limit tool, we obtain the Riordan group in its infinite and bi-infinite representations from groups of finite Riordan matrices. We define two different reflections for bi-infinite Riordan matrices. Employing these definitions, we give answers to the problems $D^{\perp} = D$ and $D^{\diamond} = D$ in the Riordan group. So, we describe the self-complementary and self-dual Riordan matrices.

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1. Introduction

The main tool used in this paper to relate finite, infinite and bi-infinite Riordan matrices is the inverse limit concept. This concept has been and is still being widely used in practically all branches of mathematics, sometimes under the name of projective limit. Usually it is a way to approximate objects by better behaved or widely known ones. The concept of inverse system, from which the inverse limit is derived, can be defined in any category, where the related concept of pro-category appears. An introductory text for these topics is [8]. In [2] the authors developed, from the categorical point of view, the Algebraic Topology. There, it can be found a study on inverse limit in the categories of both groups with homomorphisms and topological spaces with continuous maps.

Because of our previous works, [4,5,11,3], we feel the necessity of a deeper understanding of different reflections in bi-infinite Riordan matrices. To do that, we introduce herein the natural concept of finite Riordan matrices. In these kind of matrices, we get an internal transformation reflecting across $y = x$. This transformation makes no sense in usual infinite Riordan matrices. Both previous concepts on finite framework have been independently used in [1] to obtain sloping constructions of Riordan arrays. Using inverse sequences of finite Riordan matrices and the inverse limit concept, see Chapter VIII of [2], we are able to define essentially two different reflections in bi-infinite Riordan matrices. This allows us to reformulate some questions left open in [5]. Finally, we give answers to those problems.

The horizontal and vertical constructions that we explored in [4] have enabled us to understand these reflections. In fact, the fundamental point has been the symmetry between the sequence g found in [6] and used there for a vertical construction of a Riordan matrix and the A-sequence of Rogers [10] for horizontal construction of such object. Actually, the A-sequence is the parameter g in the inverse matrix, equivalently g is the A-sequence of the inverse matrix, see Proposition 7 in [3, p. 3615]. Both sequences are the same in many important cases, for instance, in self-complementary, self-dual and involutory Riordan matrices. In some works in progress, we are treating these latest kind of matrices from this point of view. We are also developing the natural pro-Lie group structure of the Riordan group derived from the constructions made herein.

After the preliminary Section 2, in Section 3 we obtain the groups of finite Riordan matrices $\mathcal{R}_n$ of order $n+1$ for $n = 0, 1, \cdots$ by means of natural projections from the Riordan group $\mathcal{R}$. Later, we recuperate the Riordan group as an inverse limit of these groups of finite matrices with appropriate bonding maps. We also give an internal characterization of such finite matrices.

In Section 4 we describe first the complementary and the dual, see [5], as elements in the inverse limit described in the previous section. To do that, we apply a natural reflection on finite Riordan matrices $\mathcal{R}_n$ of order $n+1$ for $n = 0, 1, \cdots$. Later, employing again the inverse limit concept for a constant inverse sequence, we achieve the bi-infinite representation of Riordan matrices from the usual infinite one. We end this section reaching this bi-infinite representation as inverse limit of sequences of groups of finite