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Interval max-plus matrix equations

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ABSTRACT

This paper deals with the solvability of interval matrix equations in max-plus algebra. Max-plus algebra is the algebraic structure in which classical addition and multiplication are replaced by $\oplus$ and $\otimes$, where $a \oplus b = \max\{a, b\}$ and $a \otimes b = a + b$.
The notation $\boldsymbol{A} \otimes X \otimes \boldsymbol{C} = \boldsymbol{B}$ represents an interval max-plus matrix equation, where $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are given interval matrices. We define four types of solvability of interval max-plus matrix equations, i.e., the tolerance, weak tolerance, left-weak tolerance, and right-weak tolerance solvability. We derive the necessary and sufficient conditions for checking each of them, whereby all can be verified in polynomial time.

1. Introduction

A great number of systems in which entities (machines, processors) work interactively and in stages, whereby in each stage all entities simultaneously produce components necessary for the work of some or all other entities in the next stage, can be represented by discrete event dynamic systems. One of the tools for description of dynamical systems is max-plus algebra, in which classical addition and multiplication operations are replaced

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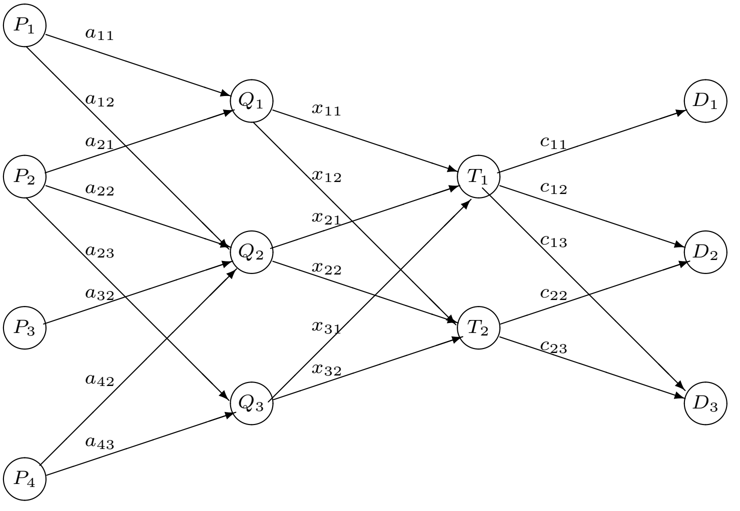

Fig. 1. Transportation system.

by maximum and addition. In the last decades, significant effort has been developed to study systems of max-plus linear equations in the form $A \otimes x = b$, where A is a matrix, b and x are vectors of compatible dimensions.

Systems of linear equations over max-plus algebra are used in several branches of applied mathematics. Among interesting real-life applications let us mention e.g. a large scale model of Dutch railway network or synchronizing traffic lights in Delfts [16].

The solvability of the systems of max-plus linear equations is quite developed. In this paper, we shall deal with two-sided matrix equations of the form $A \otimes X \otimes C = B$, where A, B, and C are given matrices of suitable sizes a X is an unknown matrix. In the following example we will show one of possible applications.

Example 1.1. Let us consider a situation, in which passengers wish to travel from places P_1, P_2, P_3, P_4 to holiday destinations D_1, D_2, D_3. Different transportation means provide transporting passengers from places P_1, P_2, P_3, P_4 to airport terminals T_1, T_2, which equip flights to destinations D_1, D_2, and D_3 (see Fig. 1). We assume that the connection between places P_i and terminals T_l is possible only via one of the check points Q_1, Q_2, Q_3. There is an arrow $(P_i\, Q_j)$ in Fig. 1 if there is a road from P_i to Q_j and there is an arrow $(T_l\, D_k)$ if terminal T_l handles passengers traveling to destination D_k ($i = 1, 2, 3, 4$; $j = 1, 2, 3$; $k = 1, 2, 3$; $l = 1, 2$). The symbols along the arrows represent the times needed for transportation and carrying out the formalities on the corresponding connections.

Denote by a_{ij} (c_{lk}) the times needed for transportation and carrying out the formalities on the connection from P_i to Q_j (from T_l to D_k). If the time needed for transportation from place Q_j to terminal T_l is x_{jl}, then the time needed for transportation from P_i to D_k via Q_j by use of terminal T_l is equal to $a_{ij} + x_{jl} + c_{lk}$.

Suppose that the time which has passengers available for the transportation from place P_i to destination D_k is denoted by b_{ik}. To ensure that all passengers from P_1 will get on time to their destinations the following equations must be satisfied: