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Equidistant subspace codes ☆

Elisa Gorla *, Alberto Ravagnani

Institut de Mathématiques, Université de Neuchâtel, Emile-Argand 11, CH-2000 Neuchâtel, Switzerland

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ABSTRACT

In this paper we study equidistant subspace codes, i.e. subspace codes with the property that each two distinct codewords have the same distance. We provide an almost complete classification of such codes, under the assumption that the cardinality of the ground field is large enough. More precisely we prove that, for most values of the parameters, an equidistant code of maximum cardinality is either a sunflower or the orthogonal of a sunflower. We also study equidistant codes with extremal parameters, and establish general properties of equidistant codes that are not sunflowers. Finally, we propose a construction of equidistant codes based on our previous construction of partial spread codes, and provide an efficient decoding algorithm.

0. Introduction

Network coding is a branch of information theory concerned with data transmission over noisy and lossy networks. A network is modeled by a directed acyclic multigraph,

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* Corresponding author.
E-mail addresses: elisa.gorla@unine.ch (E. Gorla), alberto.ravagnani@unine.ch (A. Ravagnani).

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and information travels from one or multiple sources to multiple receivers through intermediate nodes. Network coding has several applications, e.g. peer-to-peer networking, distributed storage, and patches distribution. In [1] it was proved that the information rate of a network communication may be improved employing coding at the nodes of the network, instead of simply routing the received inputs. In [15] it was shown that maximal information rate can be achieved in the multicast situation by allowing the intermediate nodes to perform linear combination of the inputs they receive, provided that the cardinality of the ground field is sufficiently large. Random linear network coding was introduced in [12], and a mathematical approach was proposed in [13] and [14], together with the definition of subspace code.

In this paper we study equidistant subspace codes, i.e., subspace codes with the property that the intersection of any pair of codewords has the same dimension. Equidistant subspace codes were shown to have relevant applications in distributed storage in [18]. In the same paper, Etzion and Raviv identify two trivial families of equidistant codes, namely sunflowers and balls. A ball is a subspace code in the Grassmannian $\mathcal{G}_q(k, n)$ of k-dimensional subspaces of $\mathbb{F}_q^n$ with the property that all the elements of the code are contained in a fixed $(k + 1)$-dimensional subspace of $\mathbb{F}_q^n$. They proceed then to study the question of when an equidistant code belongs to one of the two families. Starting from the observation that the orthogonal of a ball is a sunflower, in this paper we study the question of when an equidistant code is either a sunflower or the orthogonal of a sunflower. One of our main results is a classification of equidistant subspace codes over fields of large enough cardinality: We prove that, for most choices of the parameters, an equidistant code of maximum cardinality is either a sunflower or the orthogonal of a sunflower. In addition, for most values of the parameters the two possibilities are mutually exclusive. We also study equidistant codes for which every two distinct codewords intersect in codimension one. We show that each such code is either a sunflower of the orthogonal of a sunflower, over fields of any size and for a code of any cardinality. We also establish general properties of equidistant codes that are not sunflowers. Finally, we give a construction of asymptotically optimal equidistant codes based on the construction of partial spread codes from [11]. We then exploit the structure of our codes to design an efficient decoding algorithm for them and for their orthogonals.

The paper is organized as follows: In Section 1 we recall some definitions and results on subspace codes, equidistant codes, sunflowers, and partial spreads. In Section 2 we study equidistant codes with the property that each two distinct elements intersect in codimension one. In Section 3 we give a classification of equidistant codes for most values of k, n, and for $q \gg 0$. The classification is summarized in Theorem 27. In Section 4 we study equidistant codes that are not sunflowers. In Section 5 we give a construction of sunflower codes, and we argue that their cardinality is asymptotically optimal. In Section 6 we show how to decode them efficiently, and in Section 7 we describe their orthogonal codes and show how to decode them.