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On the spectral radius of trees with given independence number

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ABSTRACT

In the paper, we will determine the graphs with maximal spectral radius among all the trees with n vertices and independence number α for $\lceil \frac{n}{2} \rceil \leq \alpha \leq n-1$.

1. Introduction

Let $G = (V, E)$ be a simple undirected graph with n vertices. For $v \in V(G)$, we use $N_G(v)$ to denote the set of the neighbors of v and set $d_G(v) = |N_G(v)|$. For a subgraph H of G, let $N_H(v) = N_G(v) \cap V(H)$ and $d_H(v) = |N_H(v)|$ for $v \in V(G)$. If $S \subseteq V(G)$, then we will use $G[S]$ to denote the subgraph induced by S. A vertex of degree one is called a leaf. The edge incident with a leaf is known as a pendant edge. A vertex which has a leaf as its neighbor is known as a support vertex. We will use $G - xy$ to denote the

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graph that arises from G by deleting the edge $xy \in E(G)$. Similarly, $G + xy$ is a graph that arises from G by adding an edge $xy \notin E(G)$, where $x, y \in V(G)$. We will use $\alpha(G)$ to denote the independence number of G and $T_{n,\alpha}$ to denote the set of trees of order n with independence number α.

Let $A(G)$ be the adjacency matrix of a graph G. The spectral radius, $\rho(G)$, of G is the largest eigenvalue of $A(G)$. When G is connected, $A(G)$ is irreducible. By the Perron–Frobenius Theorem, the spectral radius is simple and has a unique positive eigenvector. We will refer to such an eigenvector as the Perron vector of G. Let $D(G)$ be the degree matrix of a graph G. Then $Q(G) = D(G) + A(G)$ is known as the signless Laplacian matrix of G. The signless Laplacian spectral radius of G, denoted by $\lambda(G)$, is the largest eigenvalue of $Q(G)$.

The investigation on the spectral radius of graphs is an important topic in the theory of graph spectra. Recently, the problem concerning graphs with maximal or minimal spectral radius of a given class of graphs has been studied extensively (see [1–9]). Xu et al. [9] determined the graphs with minimal spectral radius among the graphs with independence number $\alpha \in \{1, 2, \lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} \rceil + 1, n-3, n-2, n-1\}$. In addition, the relationship between spectral radius and independence number has been studied by many authors (see for examples [1,2,7]). In this paper, trees with maximal spectral radius in $T_{n,\alpha}$ for $\lceil \frac{n}{2} \rceil \leq \alpha \leq n-1$ are characterized.

2. Main results

Firstly, we cite a lemma which will be used in our proof.

Lemma 1. *(See [8].) Let u, v be two vertices of the connected graph G. Suppose $v_1, v_2, \cdots, v_s$ $(1 \leq s \leq d_v)$ are some vertices of $N_G(v) \setminus N_G(u)$ and $x = (x_1, x_2, \cdots, x_n)^T$ is the Perron vector of G, where x_i corresponds to the vertex v_i $(1 \leq i \leq n)$. Let G^* be the graph obtained from G by deleting the edges (v, v_i) and adding the edges (u, v_i) $(1 \leq i \leq s)$. If $x_u \geq x_v$, then $\rho(G) < \rho(G^*)$.*

Let $T_{n,\alpha}$ be the set of trees of order n with independence number α. Denote $\alpha = n-e$. Since $\lceil \frac{n}{2} \rceil \leq \alpha = n-e \leq n-1$, we have $1 \leq e \leq \lfloor \frac{n}{2} \rfloor$. If $e = 1$, then $T_{n,n-1} = \{K_{1,n-1}\}$, where $K_{1,n-1}$ is the star of order n. Hence we will assume that $e \geq 2$ in the following discussion.

Let $S^1_{n,n-e}$ be a set of trees on n vertices obtained from the star $K_{1,e-1}$ ($V(K_{1,e-1}) = \{v_0, v_1, \ldots, v_{e-1}\}$ and $d_{K_{1,e-1}}(v_0) = e-1, d_{K_{1,e-1}}(v_1) = \cdots = d_{K_{1,e-1}}(v_{e-1}) = 1$) by attaching at least one pendant edge to each vertex of $\{v_0, v_1, \ldots, v_{e-1}\}$ so that the total number of the leaves in $S^1_{n,n-e}$ is $n-e$. Let $S^2_{n,n-e}$ be a set of trees on n vertices obtained from the star $K_{1,e}$ ($V(K_{1,e}) = \{v_0, v_1, \ldots, v_e\}$ and $d_{K_{1,e}}(v_0) = e, d_{K_{1,e}}(v_1) = \cdots = d_{K_{1,e}}(v_e) = 1$) by attaching at least one pendant edge to each vertex of $\{v_1, \cdots, v_e\}$ so that the total number of the leaves in $S^2_{n,n-e}$ is $n-e-1$, where $e \neq \frac{n}{2}$. Notice that $S^1_{n,n-e}, S^2_{n,n-e} \subseteq T_{n,n-e}$ (see Fig. 1).