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Forward stable eigenvalue decomposition of rank-one modifications of diagonal matrices

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ABSTRACT

We present a new algorithm for solving an eigenvalue problem for a real symmetric matrix which is a rank-one modification of a diagonal matrix. The algorithm computes each eigenvalue and all components of the corresponding eigenvector with high relative accuracy in $O(n)$ operations. The algorithm is based on a shift-and-invert approach. Only a single element of the inverse of the shifted matrix eventually needs to be computed with double the working precision. Each eigenvalue and the corresponding eigenvector can be computed separately, which makes the algorithm adaptable for parallel computing. Our results extend to the complex Hermitian case. The algorithm is similar to the algorithm for solving the eigenvalue problem for real symmetric arrowhead matrices from N. Jakovčević Stor et al. (2015) [16].

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1. Introduction and preliminaries

In this paper we consider the eigenvalue problem for an $n \times n$ real symmetric matrix A of the form

$$A = D + \rho z z^T, \tag{1}$$

where

$$D = \mathrm{diag}(d_1, d_2, \ldots, d_n)$$

is a diagonal matrix of order n,

$$z = \begin{bmatrix} \zeta_1 & \zeta_2 & \cdots & \zeta_n \end{bmatrix}^T$$

is a vector and $\rho \neq 0$ is a scalar. Notice that A is a rank-one modification of a diagonal matrix. Subsequently, we shall refer to such matrices as "diagonal-plus-rank-one" (DPR1) matrices. DPR1 matrices arise, for example, in solving symmetric real tridiagonal eigenvalue problems with the divide-and-conquer method [6], [9], [13], [25, Sections 3.2.1 and 3.2.2], [26, Section III.10].

Without loss of generality, we make the following assumptions:

- $\rho > 0$ (otherwise we consider the matrix $A = -D - \rho z z^T$),
- A is irreducible, that is, $\zeta_i \neq 0$, $i = 1, \ldots, n$, and $d_i \neq d_j$, for all $i \neq j$, $i, j = 1, \ldots, n$, and
- the diagonal elements of D are decreasingly ordered,

$$d_1 > d_2 > \cdots > d_n. \tag{2}$$

Indeed, if $\zeta_i = 0$ for some i, then the diagonal element d_i is an eigenvalue whose corresponding eigenvector is the i-th unit vector, and if $d_i = d_j$, then d_i is an eigenvalue of the matrix A (we can reduce the size of the problem by annihilating ζ_j with a Givens rotation in the (i, j)-plane). Ordering of the diagonal elements of D is attained by symmetric row and column pivoting.

Let

$$A = V \Lambda V^T$$

be the eigenvalue decomposition of A, where

$$\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$$

is a diagonal matrix whose diagonal elements are the eigenvalues of A, and

$$V = \begin{bmatrix} v_1 & \cdots & v_n \end{bmatrix}$$

is an orthonormal matrix whose columns are the corresponding eigenvectors.