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Characterizing robust local error bounds for linear inequality systems under data uncertainty

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ABSTRACT

We present necessary and sufficient conditions for the existence of robust local error bounds for linear inequality system in the face of data uncertainty where the uncertain data belong to a prescribed compact uncertainty set. The robust local error bound for an uncertain system is defined in terms of the existence of local error bound for its robust counterpart, where the uncertain linear inequality is enforced for every possible value of the data in the uncertainty set. Some of these conditions are expressed using the projection of the origin and others are given by way of normal cones at the boundary points of the solution set. In the case of commonly used interval data uncertainty, we show that a qualification condition completely characterizes the existence of robust local error bounds.

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1. Introduction

Consider an *uncertain linear inequality* system

$$x \in \mathbb{R}^n, \quad a_j^\top x - b_j \leq 0, \quad j = 1, \ldots, p, \tag{SU}$$

where the data $a_j \in \mathbb{R}^n$ and $b_j \in \mathbb{R}$, $j = 1, \ldots, p$, are *uncertain*. Following the deterministic approach that is employed in Robust Optimization [2,7,8,15,16] to treat data uncertainty in optimization, we assume that the uncertain data $a_j \in \mathbb{R}^n$ and $b_j \in \mathbb{R}$, $j = 1, \ldots, p$, belong to the prescribed *uncertainty* sets U_j, $j = 1, \ldots, p$, which are assumed to be *compact* subsets in the space $\mathbb{R}^{n+1}$.

The uncertain linear inequality system (SU) is said to admit a *robust local error bound* (resp. *robust global error bound*) whenever its *robust* counterpart,

$$x \in \mathbb{R}^n, \quad a_j^\top x - b_j \leq 0, \quad \forall (a_j, b_j) \in U_j, \quad j = 1, \ldots, p, \tag{SR}$$

has a local error bound (resp. global error bound). Note that in the robust counterpart (SR) the uncertain inequalities are enforced for every possible value of the data within the uncertainty set U_j, $j = 1, \ldots, p$. The robust counterpart (SR) is a linear semi-infinite system, which has been extensively studied in the literature (see [3,4,7–10]).

Inspired by the work of Hoffman [12], a great deal of attention has been focussed on developing conditions for the existence of local error bounds for various systems of inequality systems (see e.g., [1,18–20,22,25,26,28]). These results assume perfect information (that is, accurate values for the inequality system parameters), despite the reality that such precise knowledge is rarely available in practice for real-world inequality systems. The system parameters are often uncertain (that is, they are not known exactly) due to estimation errors, prediction errors or lack of information. A small uncertainty in the data can affect the quality of the error bound of the system. Such an impact on solutions to optimization problems has already been demonstrated for linear programming problems (see [2]).

Recently, dual conditions have been presented for the existence of robust global error bounds for linear inequality systems under parameter uncertainty [5]. The purpose of the present paper is to develop characterizations for robust local error bounds for linear inequality systems (SU) in the face of parameter uncertainty.

We make the following important contributions to the topic area of error bounds for inequality systems:

(i) We provide first a characterization for the existence of *robust local error bounds* for the uncertain linear inequality system (SU) in terms of the projections of the origin. We provide a short proof by employing known results of local error bounds for convex inequality systems [25].

(ii) We then present a new characterization for the existence of *robust local error bounds* for the system (SU) in terms of normal cones at the boundary points of the solution