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Modular adjacency algebras of dual polar schemes

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ABSTRACT

We can define the adjacency algebra of an association scheme over an arbitrary field. It is not always semisimple over a field of positive characteristic. The structures of adjacency algebras over a field of positive characteristic have not been sufficiently studied.

In this paper, we consider the structures of adjacency algebras of dual polar schemes over a field of positive characteristic and determine them when their algebras are local algebras.

1. Introduction

An adjacency algebra of an association scheme over a field of characteristic zero is called the Bose–Mesner algebra. The Bose–Mesner algebra is always semisimple. Many researchers have studied this case and there are many results [2,3]. An adjacency algebra of an association scheme over a field of positive characteristic is called a modular adjacency algebra. Hanaki and Yoshikawa determined the structure of the modular adjacency algebras and the modular standard modules of association schemes of class 2 [9]. Using modular standard modules, they provided more detailed classification than using

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parameters of strongly regular graphs. This indicates that structures of the modular standard modules of association schemes provide more detailed characterization than parameters of association schemes. In order to determine the structure of the modular standard modules, we first need to obtain the structure of the modular adjacency algebras. However, the structures of adjacency algebras over fields of positive characteristic have not been sufficiently studied [8,9,11–13,17].

In this paper, we consider the structure of modular adjacency algebras of all types of dual polar schemes and determine the structure of the modular adjacency algebras of these schemes when their algebras are local algebras. The locality of algebras is characterized by their parameters in Section 4. One of our main theorems is the following.

Theorem 1. *Let F be a field of odd characteristic p, r be an odd prime power which is not divided by p and $\mathfrak{A}_d$ be a dual polar scheme on $[{}^2A_{2d}(r)]$. If $F\mathfrak{A}_d$ is a local algebra, then*

$$F\mathfrak{A}_d \cong P/W_d,$$

where P is a quotient of a polynomial ring $F[X_1, X_2, \ldots]/(X_1^p, X_2^p, \ldots)$ and W_d is an ideal generated by monomials of P such that their weights are greater than d.

These structures of modular adjacency algebras of dual polar schemes on $[{}^2A_{2d}(r)]$ decide other structures.

Theorem 2. *Let F be a field of odd characteristic p, q be an odd prime power which is not divided by p and $\mathfrak{C}_{2d'+1}$ be a dual polar scheme on $[C_{2d'+1}(q)]$. If $F\mathfrak{C}_{2d'+1}$ is a local algebra, then*

$$F\mathfrak{C}_{2d'+1} \cong P/W_{d'} \otimes P/W_1.$$

For dual polar schemes on the rest of types, we determined the structures of their modular adjacency algebras using isomorphisms or epimorphisms.

The modular adjacency algebra of Hamming scheme $H(n, p)$ over a field of characteristic p is also a local algebra [17], because the modular adjacency algebra of an association scheme with a prime power order is a local algebra over a field of characteristic the prime [8]. However the orders of our dual polar graphs are not prime powers. Therefore our situation is different from the modular adjacency algebra of Hamming scheme.

2. Preliminaries

Let X be a finite set with cardinality n. We define $R_0, \ldots, R_d$ as symmetric binary relation on X. The i-th adjacency matrix A_i is defined to be the matrix indexed by X