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Maps preserving the local spectrum of skew-product of operators

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ABSTRACT

Let $\mathscr{H}$ and $\mathscr{K}$ be infinite dimensional complex Hilbert spaces and let $\mathscr{B}(\mathscr{H})$ be the algebra of all bounded linear operators on $\mathscr{H}$. Let $\sigma_T(h)$ denote the local spectrum of an operator $T \in \mathscr{B}(\mathscr{H})$ at any vector $h \in \mathscr{H}$, and fix two nonzero vectors $h_0 \in \mathscr{H}$ and $k_0 \in \mathscr{K}$. We show that if a map $\varphi : \mathscr{B}(\mathscr{H}) \to \mathscr{B}(\mathscr{K})$ has a range containing all operators of rank at most two and satisfies

$$\sigma_{TS^*}(h_0) = \sigma_{\varphi(T)\varphi(S)^*}(k_0)$$

for all $T,\ S \in \mathscr{B}(\mathscr{H})$, then there exist two unitary operators U and V in $\mathscr{B}(\mathscr{H}, \mathscr{K})$ such that $Uh_0 = \alpha k_0$ for some nonzero $\alpha \in \mathbb{C}$ and $\varphi(T) = UTV^*$ for all $T \in \mathscr{B}(\mathscr{H})$. We also described maps $\varphi : \mathscr{B}(\mathscr{H}) \to \mathscr{B}(\mathscr{K})$ satisfying

$$\sigma_{TS^*T}(h_0) = \sigma_{\varphi(T)\varphi(S)^*\varphi(T)}(k_0)$$

for all $T,\ S \in \mathscr{B}(\mathscr{H})$, and with the range containing all operators of rank at most four.

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1. Introduction

Several results on linear or additive preserver problems have been extended to the setting of nonlinear preservers, and, in many cases, their extensions demonstrated to be nontrivial. In particular, the problem of characterizing maps preserving certain functions, subsets, relations and properties of product of matrices or operators has attracted the attention of several authors; see for instance [3,11,14,15,20,17,21] and the references therein. Motivated by problems concerning local automorphisms, L. Molnár characterized, in [17], maps preserving different spectra of operator or matrix products, and his results have been extended by several authors. In [15], J. Hou and Q.H. Di described, in particular, maps preserving the numerical range of products of Hilbert space operators. In [20], maps preserving the nilpotency of operator products are characterized. T. Miura and D. Honma gave in [16] a generalization of peripherally-multiplicative surjections between standard operator algebras. In [2], G. An and J. Hou described multiplicative maps φ on $\mathscr{B}(\mathscr{H})$, the algebra of all bounded linear operators on a complex Hilbert space $\mathscr{H}$, that satisfy $T^*S = 0$ if and only if $\varphi(T)^*\varphi(S) = 0$ for all $S,\ T \in \mathscr{B}(\mathscr{H})$. While in [22], W. Zhang and J. Hou characterized maps preserving peripheral spectrum of Jordan semi-triple product TS^*T of operators.

The problem of characterizing linear or additive maps on $\mathscr{B}(X)$, the algebra of all bounded linear operators on a complex Banach space X, preserving local spectra was initiated by A. Bourhim and T.J. Ransford in [9], and continued by several authors; see for instance [8,10–13] and the references therein. In [10], J. Bračič and V. Müller characterized surjective linear and continuous mappings on $\mathscr{B}(X)$ preserving the local spectrum and local spectral radius at a fixed nonzero vector x_0 of X, and thus extending the main results of [8] to infinite-dimensional Banach spaces. While in [13], C. Costara showed, in particular, that every surjective linear map on $\mathscr{B}(X)$ preserving the local spectral radius at a fixed nonzero vector of X is automatically continuous. Furthermore, in [12], C. Costara described surjective linear maps on $\mathscr{B}(X)$ which preserve operators of local spectral radius zero at points of X. His result has been extended in [4] to the nonlinear setting where A. Bourhim and J. Mashreghi gave a complete description of surjective (not necessarily linear) maps on $\mathscr{B}(X)$ preserving the difference of operators with local spectral radius zero at points of X. In [5], A. Bourhim and J. Mashreghi showed that surjective map φ on $\mathscr{B}(X)$ preserving the local spectrum of product of operators at a fixed nonzero vector $x_0 \in X$ if and only if φ is a positive or negative multiple automorphism and x_0 is an eigenvector of the intertwining operator. In [6], they completely describe the form of surjective maps on $\mathscr{B}(X)$ preserving the local spectrum of triple product of operators at a nonzero fixed vector of X. To have a good chronological handling of the preserving problems, see the excellent survey by A. Bourhim and J. Mashreghi [7].

Let $\mathscr{H}$ and $\mathscr{K}$ be two infinite-dimensional complex Hilbert spaces. Let $\mathscr{B}(\mathscr{H})$ be the algebra of all bounded linear operators on $\mathscr{H}$. In this paper, we follow the same path of studies by considering general local spectra preservers, and characterize maps