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A reformulation of augmented basic interpolation problem and an application to $\mathscr{H}_\infty$ control

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ABSTRACT

In this paper we present a new formulation of the augmented basic interpolation problem (aBIP) with rational matrices, in terms of the stability of four rational matrices, so that the aBIP transforms into a purely linear-algebraic problem. Actually, the existing interpolation condition, given by an integral, is replaced by stability of a rational matrix.
The new condition is applied to the $\mathscr{H}_\infty$ optimal control of one-block plants having imaginary axis invariant zeros. A new parameter in the parametrization of $\mathscr{H}_\infty$ optimal controllers is revealed, which is given in terms of the derivative of the closed-loop system at the imaginary axis invariant zeros of the plant. The $\mathscr{H}_\infty$ optimal control algorithm is illustrated by an example, and compared to the existing algorithms.

1. Introduction

Interpolation is one of the most powerful mathematical methods. Its applications are wide, examples are system identification and modeling [19,26], model reduction [5,9,11–13], and circuits and systems [20]. The latter paper presents a wide review of

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applications of interpolation. Its distinguished applications are in control theory and practice (Part VI of [1], [14], [16], and [24]).

Almost all applications can include interpolation points on the boundary of the stability region, i.e. the imaginary axis or the unit circle in the complex plane (Section 21.4 of [1], [10], [14]). Also, the applications usually require an optimal interpolant, which means that the corresponding Pick matrix is singular. Multiple interpolation points appear in system identification, when there are multiple measurement data [26]. Therefore, a very general interpolation problem is needed in applications.

One of the most general interpolation problems, which includes all of the above mentioned cases, is the augmented basic interpolation problem (aBIP) (see [8] for the problem statement, see [3,4,6–8,23] for some results, and see [7] for some examples, which illustrate the application of the aBIP to bitangential interpolation).

Formulation of aBIP. Let $A \in \mathbb{C}^{n\times n}$, $C_1 \in \mathbb{C}^{p\times n}$, $C_2 \in \mathbb{C}^{q\times n}$ and $R \in \mathbb{C}^{n\times n}$ be given matrices that satisfy the following Lyapunov equation

$$A^*R + RA + C^*JC = 0\ , \quad C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}, \quad J = \begin{bmatrix} I_p & 0 \\ 0 & -I_q \end{bmatrix}. \tag{1.1}$$

Denote $\boldsymbol{\Omega}(s) = (sI - A)^{-1}$.

The aBIP is to find a $p \times q$ dimensional rational matrix rm $\boldsymbol{U}(s)$ satisfying $\|\boldsymbol{U}\|_\infty \le 1$ such that:

(CI) The rm $\boldsymbol{U}$ is stable,

(CII) The rm $[I_p, -\boldsymbol{U}]C\boldsymbol{\Omega}$ is stable,

(CIII) The rm $\boldsymbol{\Omega}^{\#}C^*\begin{bmatrix} -\boldsymbol{U} \\ I_q \end{bmatrix}$ is stable,

(CIV) $\dfrac{1}{2\pi \mathrm{j}}\displaystyle\int_{-\mathrm{j}\infty}^{\mathrm{j}\infty} \boldsymbol{\Omega}^{\#}C^*\begin{bmatrix} I_p & -\boldsymbol{U} \\ -\boldsymbol{U}^{\#} & I_q \end{bmatrix} C\boldsymbol{\Omega}\mathrm{d}s = R.$

The purpose of this paper is to reformulate the aBIP so that the integral condition (CIV) is replaced by a condition expressed in terms of the stability of a rational matrix, and to show how the new formulation can be applied to an optimal $\mathscr{H}_\infty$ control.

Before we present technical details on the aBIP, remarks on the notation are in order.

Remarks on the notation. The abbreviation rm means rational matrix. Matrices are denoted by upper-case symbols, and vectors and scalars are denoted by lower-case symbols. By j we denote the imaginary unit. All functions of s are rational with complex coefficients, and will be written bold-faced, and if not ambiguous, without the argument. Poles and zeros (including poles and zeros at infinity) of an rm are defined through its McMillan form. If an rm is without poles in $\Re[s] \ge 0$ (in the complex plane $\mathbb{C}$), then we say that it is stable. If $\boldsymbol{U}$ is a stable rm, the distance to instability is the distance between the imaginary axis and the closest to imaginary axis pole of $\boldsymbol{U}$. An rm is proper if it has no poles at infinity. By the superscripts $^{\mathrm{T}}$ and * we denote transpose