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On the construction of graphs determined by their generalized characteristic polynomials ☆

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ABSTRACT

Given a graph G with adjacency matrix A_G, the generalized characteristic polynomial of G is defined as $\phi_G = \phi_G(\lambda, t) = \det(\lambda I - (A_G - tD_G))$ (D_G is the degree diagonal matrix). G is said to be *determined by generalized characteristic polynomial* ϕ (ϕ-DS for short) if for any graph H, $\phi(G) = \phi(H)$ implies that H is isomorphic to G.

In this paper, we show that a connected graph G is ϕ-DS if and only if the subdivision graph G_S is ϕ-DS. This gives a simple method to construct large ϕ-DS graph from smaller ones. Some further generalization of this result as wells as some non-trivial examples of constructing ϕ-DS graphs are also provided.

1. Introduction

In this paper, we are concerned with finite, undirected and (multi)-graphs without loops. Let $G = (V(G), E(G))$ be a graph with the vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and the edge set $E(G) = \{e_1, e_2, \ldots, e_m\}$. Let A_G be the adjacency matrix of G and D_G

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be the degree diagonal matrix. *The generalized characteristic polynomial* of G, introduced by Cvetković et al. [2], is defined as $\phi_G(\lambda, t) = \det(\lambda I - (A_G - tD_G))$ (or ϕ_G or simply ϕ if no confusion arises).

The polynomial $\phi_G(\lambda, t)$ generalizes some well known characteristic polynomials of graphs G (see e.g. [2]). It has also an elegant combinatorial interpretation as being equivalent to the Bartholdi zeta function – a generalization of the Ihara–Selberg zeta function, of graph G (see [1,6,7]).

Two graphs G and H are cospectral if they share the same spectrum. A graph G is said to be *determined by the spectrum* (DS for short), if any graph that is cospectral with G is isomorphic to G (of course, the spectrum concerned should be specified). So if G is determined by the adjacency spectrum, we say G is A-DS, also we have L-DS for Laplacian spectrum, Q-DS for signless Laplacian spectrum, and so on. In particular, two graphs G and H are ϕ-cospectral if $\phi_G = \phi_H$. A graph G is said to *be determined by* ϕ_G (ϕ-DS for short), if any graph that is ϕ-cospectral with G is isomorphic to G.

In [10], Wang et al. addressed the problem of characterizing graphs by their generalized characteristic polynomials. The problem is essentially equivalent to the spectral characterization of graphs w.r.t. a family of matrices $A_G - tD_G$ ($t \in \mathbf{R}$), simultaneously. We refer the reader to [8,9] for some surveys on the topic of spectral characterization of graphs.

In [10,11], the authors give some invariants of graphs for the generalized characteristic polynomial ϕ, based on which they are able to show that some particular family of graphs are determined by ϕ (such as the rose graphs, the wheel graphs and some families of bi-regular graphs).

In this paper, we shall continue this line of research by giving more ϕ-DS graphs. The main result of the paper shows that a connected graph G is ϕ-DS if and only if its subdivision graph G_S is ϕ-DS. This gives a simple method to construct large ϕ-DS graph from smaller ones. Some further generalization of this result as wells as some non-trivial examples of constructing ϕ-DS graphs are also provided.

It is worthwhile to notice that in a recent paper [4], the authors studied a similar topic of distinguishing graphs with zeta functions and generalized spectra. They showed that large classes of cospectral graphs can be distinguished with zeta functions and enumerate graphs distinguished by zeta functions on $n \leq 11$ vertices. They conjectured, among others, that almost all graphs which are not determined by their spectrum are determined by zeta functions. And hence, the generalized characteristic polynomial of graphs (and zeta functions) seems to be a promising tool to distinguish various graphs.

The rest of the paper is organized as follows. In Section 2, we show that for any (multi)-graph G, whether G is determined by ϕ_G is equivalent to whether the corresponding subdivision graph G_S is determined by ϕ_G. In Section 3, we give an extension of subdivision graph, and show that graph G is ϕ-DS iff the corresponding well defined generalized subdivision graph G_Γ is ϕ-DS. In Section 4, some applications are given by constructing infinity families of ϕ-DS graphs from A-DS multi-graphs. Conclusions are given in Section 5.