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Nonnegative persymmetric matrices with prescribed elementary divisors ☆

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ABSTRACT

The *nonnegative inverse elementary divisors problem* (*NIEDP*) is the problem of finding conditions for the existence of an $n \times n$ entrywise nonnegative matrix A with prescribed elementary divisors. We consider the case in which the solution matrix A is required to be persymmetric. Persymmetric matrices are common in physical sciences and engineering. They arise, for instance, in the control of mechanical and electric vibrations. In this paper, we solve the *NIEDP* for $n \times n$ matrices assuming that (i) there exists a partition of the given list $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$ in sublists Λ_k, along with suitably chosen Perron eigenvalues, which are realizable by nonnegative matrices A_k with certain of the prescribed elementary divisors, and (ii) a nonnegative persymmetric matrix exists with diagonal entries being the Perron eigenvalues of the matrices A_k, with certain of the prescribed elementary divisors. Our results generate an algorithmic procedure to compute the structured solution matrix.

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1. Introduction

Let $A \in \mathbb{C}^{n\times n}$ and let

$$J(A) = S^{-1}AS = \begin{bmatrix} J_{n_1(\lambda_1)} & & & \\ & J_{n_2(\lambda_2)} & & \\ & & \ddots & \\ & & & J_{n_k(\lambda_k)} \end{bmatrix}$$

be the *Jordan canonical form* of A (hereafter *JCF* of A). The $n_i \times n_i$ submatrices

$$J_{n_i}(\lambda_i) = \begin{bmatrix} \lambda_i & 1 & & \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ & & & \lambda_i \end{bmatrix}, \quad i = 1, 2, \ldots, k$$

are called the *Jordan blocks* of $J(A)$. The *elementary divisors* of A are the polynomials $(\lambda-\lambda_i)^{n_i}$, that is, the characteristic polynomials of $J_{n_i}(\lambda_i)$, $i = 1, \ldots, k$. The *nonnegative inverse elementary divisors problem* (hereafter *NIEDP*) is the problem of determining necessary and sufficient conditions under which the polynomials $(\lambda-\lambda_1)^{n_1}, (\lambda-\lambda_2)^{n_2}, \ldots,$ $(\lambda-\lambda_k)^{n_k}$, $n_1+\cdots+n_k = n$, are the elementary divisors of an $n\times n$ nonnegative matrix A [5,6,12].

In [5,6] Minc considers the *NIEDP* modulo the *nonnegative inverse eigenvalue problem* (hereafter *NIEP*), which is the problem of finding necessary and sufficient conditions for the existence of a nonnegative matrix with prescribed spectrum. We approach the *NIEDP* in the same way. In particular, Minc showed that if A is a diagonalizable positive matrix (diagonalizable positive doubly stochastic matrix), then there exists a positive matrix B (positive doubly stochastic matrix B) with the same spectrum as A, and with any prescribed elementary divisors. In [12,15], the authors completely solve the *NIEDP* for lists of real numbers $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$ satisfying: *i*) $\lambda_1 > \lambda_2 \geq \cdots \geq \lambda_n \geq 0$, and *ii*) $\lambda_1 > 0 > \lambda_2 \geq \cdots \geq \lambda_n$, and for lists of complex numbers satisfying *iii*) $\operatorname{Re}\lambda_i < 0$, $|\operatorname{Re}\lambda_i| \geq |\operatorname{Im}\lambda_i|$, $i = 2, \ldots, n$. Sufficient conditions are also given in [12,15] for the general case.

A matrix $A = (a_{ij})_{i,j=1}^n$ is said to have *constant row sums* if all its rows add up to the same constant γ, i.e. $\sum\limits_{j=1}^{n} a_{ij} = \gamma$, $i = 1, \ldots, n$. The set of all matrices with constant row sums equal to γ is denoted by $\mathcal{CS}_\gamma$. It is clear that $\mathbf{e} = (1, 1, \ldots, 1)^T$ is an eigenvector of any matrix $A \in \mathcal{CS}_\gamma$, corresponding to the eigenvalue γ. A nonnegative matrix A is called *stochastic* if $A \in \mathcal{CS}_1$ and is called *doubly stochastic* if $A, A^T \in \mathcal{CS}_1$. A matrix $A \in \mathcal{CS}_{\lambda_1}$ or $A, A^T \in \mathcal{CS}_{\lambda_1}$, is called *generalized stochastic* or *generalized doubly stochastic*, respectively. The relevance of the real matrices with constant row sums is due