Linear Algebra and its Applications 484 (2015) 154–174

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Necessary and sufficient conditions for unified optimality of interval linear program in the general form

Haohao Li

School of Mathematics and Statistics, Zhejiang University of Finance and Economics, Hangzhou, 310018, China

A R T I C L E   I N F O

Article history:
Received 17 December 2014
Accepted 24 June 2015
Available online 8 July 2015
Submitted by R. Brualdi

MSC:
65G40

Keywords:
Interval linear programming
Unified optimal solutions
Unified feasible solutions
KKT conditions

A B S T R A C T

This paper considers optimal solutions of general interval linear programming problems. The most general concepts of optimal solutions are introduced in a unified framework. The existing optimal solution concepts of interval linear program such as weak and strong optimal solutions are all the special cases in this framework. Necessary and sufficient conditions for checking a class of optimality are developed according to the KKT conditions. Also, the comparison among several conclusions using KKT conditions and tangent core technique in various forms of interval linear programming is proposed.

1. Introduction

The interval linear programming (IvLP) problems whose parameters can vary within some given intervals have been investigated by many authors, see e.g. [1–8], among others. One of the main difficulties while dealing with interval systems and interval linear programs is how to understand the concepts of solutions and optimal solutions.

E-mail address: hhli@zufe.edu.cn.

http://dx.doi.org/10.1016/j.laa.2015.06.029
0024-3795/

For interval equations, Shary [9,10] first defined the concept of AE solutions which is characterized by ∀∃-quantifications. Hladík [11] utilized the concept of AE solutions for interval inequalities. Recently, the concepts of quantified optimal solutions to IvLP were introduced in [12–15], and some necessary and sufficient conditions were proposed for checking such new optimality. These researches considered IvLP with different forms of constraint conditions separately, since the interval systems cannot be transformed conveniently to each other due to dependency. In addition, the techniques used in these references are based on the theory of tangent cones whose theoretical foundation is relatively complicated.

In this paper, we propose a unified optimal solution concept of interval linear program in the general form. The concepts of optimal solutions of IvLP discussed before are all special cases in this new framework. Furthermore, the techniques used in this paper for establishing the necessary and sufficient conditions for optimality are based on KKT conditions, which are relatively simple and familiar than those used in the previously published papers.

2. Unified optimal solution concept of interval linear program

Following notations from [5], an interval matrix is defined as

$$\mathbf{A} = [\underline{A}, \overline{A}] = \{A \in \mathbb{R}^{m\times n} | \underline{A} \leq A \leq \overline{A}\},$$

where $\underline{A}, \overline{A} \in \mathbb{R}^{m\times n}$, and $\underline{A} \leq \overline{A}$. Similarly, we define an interval vector as an one column interval matrix

$$\mathbf{b} = [\underline{b}, \bar{b}] = \{b \in \mathbb{R}^{m} | \underline{b} \leq b \leq \bar{b}\},$$

where $\underline{b}, \bar{b} \in \mathbb{R}^m$, and $\underline{b} \leq \bar{b}$. The set of all m-by-n interval matrices will be denoted by $\mathbb{IR}^{m\times n}$ and the set of all m-dimensional interval vectors by $\mathbb{IR}^m$.

Denote by A_c and A_Δ the center and radius matrices given by

$$A_c = \frac{1}{2}(\underline{A} + \overline{A}), A_\Delta = \frac{1}{2}(\overline{A} - \underline{A})$$

respectively. Then $\mathbf{A} = [A_c - A_\Delta, A_c + A_\Delta]$. Similarly, the center and radius vectors are defined as

$$b_c = \frac{1}{2}(\underline{b} + \bar{b}), b_\Delta = \frac{1}{2}(\bar{b} - \underline{b})$$

respectively. Then $\mathbf{b} = [b_c - b_\Delta, b_c + b_\Delta]$.

Let Y_m be the set of all $\{-1, 1\}$ m-dimensional vectors, i.e.

$$Y_m = \{y \in \mathbb{R}^m | \mid y \mid = e\},$$

where $e = (1, \cdots, 1)^T$ is the m-dimensional vector of all 1's. For a given $y \in Y_m$, let