

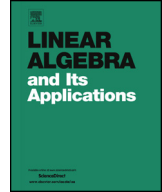


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Leonard triples, the Racah algebra, and some distance-regular graphs of Racah type



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ABSTRACT

By a Leonard triple, we mean a triple of diagonalizable operators on a finite-dimensional vector space such that for each operator, there is an ordering of an eigenbasis for the selected operator with respect to which the other two operators are irreducible tridiagonal.

Let $\mathbb{C}$ denote the field of complex numbers and let D denote an integer at least 3. Let $\frac{1}{2}H''(2D+1, 2)$ denote the halved graph of the $(2D+1)$ -cube with respect to the original P -polynomial structure $R_0, R_1, \dots, R_D$ and another Q -polynomial structure $E_0, E_2, E_4, \dots, E_3, E_1$ in terms of the original ones. Let $\frac{1}{2}\bar{H}(4D, 2)$ denote the folded halved graph of the $4D$ -cube and let $\frac{1}{2}\bar{H}(4D+2, 2)$ denote the folded halved graph of the $(4D+2)$ -cube. Note that they are all distance-regular graphs of Racah type.

In this paper we consider the relations between the above three graphs and the Leonard triples or the Racah algebra over $\mathbb{C}$. Our results are described as follows.

1. Fix a vertex of $\frac{1}{2}H''(2D+1, 2)$ and let T_1 denote the corresponding Terwilliger algebra with respect to this vertex. We first construct three elements $\mathfrak{U}_1, \mathfrak{U}_1^*$ and $\mathfrak{U}_1^\varepsilon$ of T_1 . Then we show that the triple $\mathfrak{U}_1, \mathfrak{U}_1^*, \mathfrak{U}_1^\varepsilon$ acts on each irreducible T_1 -module as a Leonard triple. Moreover, let $\mathfrak{R}_1$ be a Racah algebra with its generators and real parameters satisfying certain conditions. We display a $\mathbb{C}$ -algebra homomorphism from $\mathfrak{R}_1$ to T_1 .

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2. Fix a vertex of $\frac{1}{2}\bar{H}(4D, 2)$ and let T_2 denote the Terwilliger algebra of $\frac{1}{2}\bar{H}(4D, 2)$ with respect to this vertex. We construct three elements $\mathfrak{U}_2, \mathfrak{U}_2^*, \mathfrak{U}_2^\xi$ of T_2 and show that the triple $\mathfrak{U}_2, \mathfrak{U}_2^*, \mathfrak{U}_2^\xi$ not only acts on each irreducible T_2 -module as a Leonard triple but also satisfies some very appealing equations. Moreover, let W denote an irreducible T_2 -module with type ψ and let $\mathfrak{K}_\psi$ be a Racah algebra with respect to ψ . Then there exists a $\mathfrak{K}_\psi$ -module structure on W .
3. Fix a vertex of $\frac{1}{2}\bar{H}(4D + 2, 2)$ and let T_3 denote the Terwilliger algebra of $\frac{1}{2}\bar{H}(4D + 2, 2)$ with respect to this vertex. We construct three elements $\mathfrak{U}_3, \mathfrak{U}_3^*, \mathfrak{U}_3^\xi$ of T_3 and show that the triple $\mathfrak{U}_3, \mathfrak{U}_3^*, \mathfrak{U}_3^\xi$ not only acts on each irreducible T_3 -module as a Leonard triple but also satisfies some very appealing equations. Moreover, let W denote an irreducible T_3 -module with auxiliary parameter e and let $\mathfrak{K}_e$ be a Racah algebra with respect to e . Then there exists a $\mathfrak{K}_e$ -module structure on W .

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1. Introduction

Leonard pairs were introduced by P. Terwilliger in [30]. By a *Leonard pair*, we mean a pair of diagonalizable operators on a finite-dimensional vector space such that for each operator, there is an ordering of an eigenbasis for the selected operator with respect to which the other operator is irreducible tridiagonal. Leonard pairs arise in connection with representation theory [19,30,31,36] and combinatorics [4,5,8,15,19,28,29,32], and help elucidate their connections to certain orthogonal polynomials [22,23].

Leonard triples were introduced by B. Curtin in [7]. By a *Leonard triple*, we mean a triple of diagonalizable operators on a finite-dimensional vector space such that for each operator, there is an ordering of an eigenbasis for the selected operator with respect to which the other two operators are irreducible tridiagonal. In addition to representation theory and combinatorics, Leonard triples are related to spin models [6], the generalized Markov problem in number theory, and the Poncelet problem in projective geometry [21].

The halved graph $\frac{1}{2}H(2D + 1, 2)$ of the $(2D + 1)$ -cube with P -polynomial structure $R_0, R_1, \dots, R_D$ and Q -polynomial structure $E_0, E_1, \dots, E_D$ has vertex set consisting of all $(2D + 1)$ -tuples of 1 's and -1 's that contain an even number of 1 's. Vertices are adjacent if they differ in exactly two coordinates. $\frac{1}{2}H(2D + 1, 2)$ has another P -polynomial structure $R_0, R_D, R_1, R_{D-1}, \dots$ and another Q -polynomial structure $E_0, E_2, E_4, \dots, E_3, E_1$ in terms of the original ones. Let $\frac{1}{2}H''(2D + 1, 2)$ denote the halved graph of the $(2D + 1)$ -cube with respect to original P -polynomial structure and the new Q -polynomial structure.

Among the various constructions of the folded halved graph $\frac{1}{2}\bar{H}(4D, 2)$ of the $4D$ -cube we may take the following. The vertex set of $\frac{1}{2}\bar{H}(4D, 2)$ consists of the $4D$ -tuples of 0 's

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