Linear Algebra and its Applications 486 (2015) 469–474

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Hyperinvariant subspaces of locally nilpotent linear transformations ☆

Pudji Astuti [a], Harald K. Wimmer [b,*]

[a] *Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Bandung 40132, Indonesia*

[b] *Mathematisches Institut, Universität Würzburg, 97074 Würzburg, Germany*

ARTICLE INFO

Article history:
Received 13 June 2015
Accepted 1 September 2015
Available online 9 September 2015
Submitted by R. Brualdi

MSC:
15A04
47A15

Keywords:
Locally nilpotent operators
Hyperinvariant subspaces
Invariant subspaces
Cyclic subspaces
Endomorphism ring
Exponent
Height

ABSTRACT

A subspace X of a vector space over a field K is hyperinvariant with respect to an endomorphism f of V if it is invariant for all endomorphisms of V that commute with f. We assume that f is locally nilpotent, that is, every $x \in V$ is annihilated by some power of f, and that V is an infinite direct sum of f-cyclic subspaces. In this note we describe the lattice of hyperinvariant subspaces of V. We extend a result of Fillmore, Herrero and Longstaff (1977) [2] to infinite dimensional spaces.

☆ The work of the first author was supported by the program "Riset dan Inovasi KK ITB" of the Institut Teknologi Bandung.

* Corresponding author.
E-mail addresses: pudji@math.itb.ac.id (P. Astuti), wimmer@mathematik.uni-wuerzburg.de (H.K. Wimmer).

http://dx.doi.org/10.1016/j.laa.2015.09.002
0024-3795/

1. Introduction

Let V be a vector space over a field K and let f be an endomorphism of V. A subspace X of V is called *hyperinvariant* (with respect of f) if it is invariant for all endomorphisms of V that commute with f (see [1, p. 227], [4, p. 305]). An endomorphism f of V is said to be *locally nilpotent* [5, p. 37] if every $x \in V$ is annihilated by some power of f. In this note we are concerned with locally nilpotent endomorphisms with the property that the underlying vector space V is an infinite direct sum of finite-dimensional f-cyclic subspaces. It is the purpose of our paper to describe the lattice of hyperinvariant subspaces of V. We extend results of Fillmore, Herrero and Longstaff [2] (see [4, Chapter 9]) to infinite-dimensional spaces.

Let t_j, $j \in \mathbb{N}$, be the dimensions of the cyclic subspaces that are direct summands of V. Then V has a decomposition

$$V = \bigoplus_{j\in\mathbb{N}} V_{t_j}, \quad V_{t_j} = \bigoplus_{\sigma\in S_j} V_{t_j\sigma} \quad \text{where} \quad V_{t_j\sigma} \cong K[s]/f^{t_j}K[s]. \tag{1.1}$$

In (1.1) the direct summands of dimension t_j are gathered together in subspaces V_{t_j}, respectively. We assume

$$t_j < t_{j+1}, \ j \in \mathbb{N}. \tag{1.2}$$

The main result of the paper is the following.

Theorem 1.1. *Suppose V is locally nilpotent with respect to f and let (1.1) and (1.2) hold. For a subspace $X \subseteq V$ the following statements are equivalent.*

(i) *X is hyperinvariant.*

(ii) *X is of the form*

$$X = \bigoplus_{j\in\mathbb{N}} f^{r_j} V_{t_j} \tag{1.3}$$

with

$$0 \le r_j \le t_j, \ j \in \mathbb{N}, \tag{1.4}$$

and

$$r_j \le r_\ell, \quad t_j - r_j \le t_\ell - r_\ell, \quad \text{if} \quad j \le \ell. \tag{1.5}$$

(iii) *We have*

$$X = \sum_{j\in\mathbb{N}} \left(\operatorname{Im} f^{r_j} \cap \operatorname{Ker} f^{t_j - r_j}\right) \tag{1.6}$$

with $(r_j)_{j\in\mathbb{N}}$ satisfying (1.4) and (1.5).