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On Farkas lemma and dimensional rigidity of bar frameworks ☆

A.Y. Alfakih

Department of Mathematics and Statistics, University of Windsor, Windsor, Ontario N9B 3P4, Canada

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ABSTRACT

We present a new semidefinite Farkas lemma involving a side constraint on the rank. This lemma is then used to refine and elaborate on a recent characterization, by Connelly and Gortler [7], of dimensional rigidity of bar frameworks.

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E-mail address: alfakih@uwindsor.ca.

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1. Introduction

The celebrated Farkas lemma is at the core of optimization theory. It underpins duality theory of linear programming, and its semidefinite version plays a key role in strong duality results of semidefinite programming. As an example of *theorems of the alternative*, Farkas lemma establishes the infeasibility of a given linear matrix inequality by exhibiting a solution for another linear matrix inequality. In this paper, we present a new semidefinite Farkas lemma (Theorem 2.2 below) involving a side constraint on the rank. This Farkas lemma is then used to refine and elaborate on a recent characterization, by Connelly and Gortler [7], of dimensional rigidity of bar frameworks.

A *bar framework* in $\mathbb{R}^r$, denoted by (G,p), is a simple connected undirected graph $G=(V,E)$ whose nodes are points $p^1,\ldots,p^n$ in $\mathbb{R}^r$; and whose edges are line segments, each joining a pair of these points. We say that (G,p) is r-dimensional if the points $p^1,\ldots,p^n$ affinely span $\mathbb{R}^r$.

Let (G,p) and (G,p') be two r-dimensional and s-dimensional frameworks in $\mathbb{R}^r$ and $\mathbb{R}^s$ respectively. Then (G,p') is *equivalent* to (G,p) if:

$$\|p'^{\,i}-p'^{\,j}\|^2=\|p^i-p^j\|^2 \quad \text{for each } \{i,j\}\in E(G), \tag{1}$$

where $\|.\|$ denotes the Euclidean norm and $E(G)$ denotes the edge set of graph G. Moreover, (G,p') is said to be *affinely equivalent* to (G,p) if (G,p') is equivalent to (G,p) and $p'^{\,i}=Ap^i+b$ for all $i=1,\ldots,n$, where A is an $r\times r$ matrix and b is a vector in $\mathbb{R}^r$. Finally, two r-dimensional frameworks (G,p) and (G,p') in $\mathbb{R}^r$ are *congruent* if:

$$\|p'^{\,i}-p'^{\,j}\|^2=\|p^i-p^j\|^2 \quad \text{for all } i,j=1,\ldots,n. \tag{2}$$

An r-dimensional framework (G,p) is said to be *dimensionally rigid* if no s-dimensional framework (G,p'), for any $s\ge r+1$, is equivalent to (G,p). On the other hand, if every s-dimensional framework (G,p'), for any s, that is equivalent to (G,p) is in fact congruent to (G,p), then framework (G,p) is said to be *universally rigid*. It turns out that dimensional rigidity and universal rigidity are closely related.

Theorem 1.1 *(Alfakih [1]). Let (G,p) be an r-dimensional bar framework on n vertices in $\mathbb{R}^r$, for some $r\le n-2$. Then (G,p) is universally rigid if and only if the following two conditions hold:*

1. *(G,p) is dimensionally rigid.*
2. *There does not exist an r-dimensional framework (G,p') in $\mathbb{R}^r$ that is affinely equivalent, but not congruent, to (G,p).*

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