

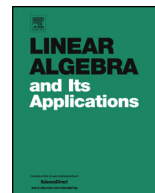


ELSEVIER

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa



On Farkas lemma and dimensional rigidity of bar frameworks[☆]



A.Y. Alfakih

Department of Mathematics and Statistics, University of Windsor, Windsor, Ontario N9B 3P4, Canada

ARTICLE INFO

Article history:

Received 1 June 2014

Accepted 28 August 2015

Available online 10 September 2015

Submitted by R. Brualdi

MSC:

90C22

90C25

52C25

05C62

Keywords:

Farkas lemma

Bar frameworks

Dimensional rigidity

Universal rigidity

Facial reduction

Semidefinite programming

Stress matrices

ABSTRACT

We present a new semidefinite Farkas lemma involving a side constraint on the rank. This lemma is then used to refine and elaborate on a recent characterization, by Connelly and Gortler [7], of dimensional rigidity of bar frameworks.

© 2015 Elsevier Inc. All rights reserved.

[☆] Research supported by the Natural Sciences and Engineering Research Council of Canada.

E-mail address: alfakih@uwindsor.ca.

1. Introduction

The celebrated Farkas lemma is at the core of optimization theory. It underpins duality theory of linear programming, and its semidefinite version plays a key role in strong duality results of semidefinite programming. As an example of *theorems of the alternative*, Farkas lemma establishes the infeasibility of a given linear matrix inequality by exhibiting a solution for another linear matrix inequality. In this paper, we present a new semidefinite Farkas lemma (Theorem 2.2 below) involving a side constraint on the rank. This Farkas lemma is then used to refine and elaborate on a recent characterization, by Connelly and Gortler [7], of dimensional rigidity of bar frameworks.

A *bar framework* in \mathbb{R}^r , denoted by (G, p) , is a simple connected undirected graph $G = (V, E)$ whose nodes are points p^1, \dots, p^n in \mathbb{R}^r ; and whose edges are line segments, each joining a pair of these points. We say that (G, p) is *r-dimensional* if the points p^1, \dots, p^n affinely span \mathbb{R}^r .

Let (G, p) and (G, p') be two *r-dimensional* and *s-dimensional* frameworks in \mathbb{R}^r and \mathbb{R}^s respectively. Then (G, p') is *equivalent* to (G, p) if:

$$\|p'^i - p'^j\|^2 = \|p^i - p^j\|^2 \quad \text{for each } \{i, j\} \in E(G), \tag{1}$$

where $\|\cdot\|$ denotes the Euclidean norm and $E(G)$ denotes the edge set of graph G . Moreover, (G, p') is said to be *affinely equivalent* to (G, p) if (G, p') is equivalent to (G, p) and $p'^i = Ap^i + b$ for all $i = 1, \dots, n$, where A is an $r \times r$ matrix and b is a vector in \mathbb{R}^r . Finally, two *r-dimensional* frameworks (G, p) and (G, p') in \mathbb{R}^r are *congruent* if:

$$\|p'^i - p'^j\|^2 = \|p^i - p^j\|^2 \quad \text{for all } i, j = 1, \dots, n. \tag{2}$$

An *r-dimensional* framework (G, p) is said to be *dimensionally rigid* if no *s-dimensional* framework (G, p') , for any $s \geq r + 1$, is equivalent to (G, p) . On the other hand, if every *s-dimensional* framework (G, p') , for any s , that is equivalent to (G, p) is in fact congruent to (G, p) , then framework (G, p) is said to be *universally rigid*. It turns out that dimensional rigidity and universal rigidity are closely related.

Theorem 1.1 (Alfakih [1]). *Let (G, p) be an r-dimensional bar framework on n vertices in \mathbb{R}^r , for some $r \leq n - 2$. Then (G, p) is universally rigid if and only if the following two conditions hold:*

1. (G, p) is dimensionally rigid.
2. There does not exist an *r-dimensional* framework (G, p') in \mathbb{R}^r that is affinely equivalent, but not congruent, to (G, p) .

Download English Version:

<https://daneshyari.com/en/article/4599017>

Download Persian Version:

<https://daneshyari.com/article/4599017>

[Daneshyari.com](https://daneshyari.com)