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Projection onto simplicial cones by Picard's method

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ABSTRACT

By using Moreau's decomposition theorem for projecting onto cones, the problem of projecting onto a simplicial cone is reduced to finding the unique solution of a nonsmooth system of equations. It is shown that Picard's method applied to the system of equations associated with the problem of projecting onto a simplicial cone generates a sequence that converges linearly to the solution of the system. Numerical experiments are presented making the comparison between Picard's and semi-smooth Newton's methods to solve the nonsmooth system associated with the problem of projecting a point onto a simplicial cone.

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1. Introduction

The interest in the subject of projection arises in several situations, having a wide range of applications in pure and applied mathematics such as convex analysis (see e.g. [1]), optimization (see e.g. [2–7]), numerical linear algebra (see e.g. [8]), statistics (see e.g. [9–11]), computer graphics (see e.g. [12]) and ordered vector spaces (see e.g. [13–18]). More specifically, the projection onto a polyhedral cone, which has as a special case the projection onto a simplicial one, is a problem of high impact on scientific community.[4] The geometric nature of this problem makes it particularly interesting and important in many areas of science and technology such as statistics (see e.g. [11]), computation (see e.g. [19]), optimization (see e.g. [20,7]) and ordered vector spaces (see e.g. [16]).

The projection onto a general simplicial cone is difficult and computationally expensive, this problem has been studied e.g. in [21,22,5,23,16,7]. It is a special convex quadratic program and its KKT optimality conditions form the linear complementarity problem (LCP) associated with it, see e.g. [24,23,7]. Therefore, the problem of projecting onto simplicial cones can be solved by active set methods [25–27,24] or any algorithms for solving LCPs, see e.g. [25,24] and special methods based on its geometry, see e.g. [23,24]. Other fashionable ways to solve this problem are based on the classical von Neumann algorithm (see e.g. the Dykstra algorithm [28,10,29]). Nevertheless, these methods are also quite expensive (see the numerical results in [20] and the remark preceding Section 6.3 in [30]).

In this paper we particularize the Moreau decomposition theorem for simplicial cones. This leads to an equivalence between the problem of projecting a point onto a simplicial cone and one of finding the unique solution of a nonsmooth system of equations. We apply Picard's method to find a unique solution of the obtained associated system. Under a mild assumption on the simplicial cone we show that the method generates a sequence that converges linearly to the solution of the associated system of equations. Numerical experiments are presented making the comparison between Picard's and semi-smooth Newton's methods for solving the nonsmooth system associated with the problem of projecting a point onto a simplicial cone.

The organization of the paper is as follows. In Section 2, some notations, basic results used in the paper and the statement of the problems that we are interested are presented, in particular, the problem of projecting onto simplicial cone. In Section 3 we present some results about projection onto simplicial cones. In Section 4 we present two different Picard's iterations for solving the problem of projecting onto simplicial cone. In Section 5 theoretical and numerical comparisons between Picard's methods and semi-smooth Newton's method for solving the problem of projecting onto simplicial cone [31] are provided. Some final remarks are made in Section 6.

[4] See the popularity of the Wikimization page Projection on Polyhedral Cone at http://www.convexoptimization.com/wikimization/index.php/Special:Popularpages.