Linear Algebra and its Applications 480 (2015) 93–106

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

On the spectral radius of a class of non-odd-bipartite even uniform hypergraphs ☆

Murad-ul-Islam Khan, Yi-Zheng Fan *

School of Mathematical Sciences, Anhui University, Hefei 230601, PR China

A R T I C L E   I N F O

Article history:
Received 13 September 2014
Accepted 4 April 2015
Available online 29 April 2015
Submitted by R. Brualdi

Keywords:
Hypergraph
Non-odd-bipartiteness
Adjacency tensor
Signless Laplacian tensor
Spectral radius

A B S T R A C T

In order to investigate the non-odd-bipartiteness of even uniform hypergraphs, starting from a simple graph G, we construct a generalized power of G, denoted by $G^{k,s}$, which is obtained from G by blowing up each vertex into a s-set and each edge into a $(k-2s)$-set, where $s \le k/2$. When $s < k/2$, $G^{k,s}$ is always odd-bipartite. We show that $G^{k,\frac{k}{2}}$ is non-odd-bipartite if and only if G is non-bipartite, and find that $G^{k,\frac{k}{2}}$ has the same adjacency (respectively, signless Laplacian) spectral radius as G. So the results involving the adjacency or signless Laplacian spectral radius of a simple graph G hold for $G^{k,\frac{k}{2}}$. In particular, we characterize the unique graph with minimum adjacency or signless Laplacian spectral radius among all non-odd-bipartite hypergraphs $G^{k,\frac{k}{2}}$ of fixed order, and prove that $\sqrt{2+\sqrt{5}}$ is the smallest limit point of the non-odd-bipartite hypergraphs $G^{k,\frac{k}{2}}$. In addition we obtain some results for the spectral radii of the weakly irreducible nonnegative tensors.

☆ Supported by the National Natural Science Foundation of China (11371028), Program for New Century Excellent Talents in University (NCET-10-0001), Scientific Research Fund for Fostering Distinguished Young Scholars of Anhui University (KJJQ1001), Academic Innovation Team of Anhui University Project (KJTD001B).

* Corresponding author.
E-mail addresses: muradulislam@foxmail.com (M.-I. Khan), fanyz@ahu.edu.cn (Y.-Z. Fan).

http://dx.doi.org/10.1016/j.laa.2015.04.005
0024-3795/

1. Introduction

Hypergraphs are a generalization of simple graphs. They are really handy to show complex relationships found in the real world. A *hypergraph* $G = (V(G), E(G))$ is a set of vertices, say $V(G) = \{v_1, v_2, \dots, v_n\}$, and a set of edges, say $E(G) = \{e_1, e_2, \dots, e_m\}$ where $e_j \subseteq V(G)$. If $|e_j| = k$ for each $j = 1, 2, \dots, m$, then G is called a *k-uniform* hypergraph. In particular, the 2-uniform hypergraphs are exactly the classical simple graphs. The *degree* d_v of a vertex $v \in V(G)$ is defined as $d_v = |\{e_j : v \in e_j \in E(G)\}|$. A *walk* W of length l in G is a sequence of alternate vertices and edges: $v_0, e_1, v_1, e_2, \dots, e_l, v_l$, where $\{v_i, v_{i+1}\} \subseteq e_i$ for $i = 0, 1, \dots, l-1$. If $v_0 = v_l$, then W is called a *circuit*. A walk in G is called a *path* if no vertices or edges are repeated. A circuit in G is called a *cycle* if no vertices or edges are repeated. The hypergraph G is said to be *connected* if every two vertices are connected by a walk. A hypergraph H is a *sub-hypergraph* of G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$, and H is a *proper sub-hypergraph* of G if $V(H) \subsetneq V(G)$ or $E(H) \subsetneq E(G)$.

In recent years spectral hypergraph theory has emerged as an important field in algebraic graph theory. Let G be a k-uniform hypergraph. The *adjacency tensor* $\mathcal{A} = \mathcal{A}(G) = (a_{i_1 i_2 \dots i_k})$ of G is a kth order n-dimensional symmetric tensor, where

$$a_{i_1 i_2 \dots i_k} = \begin{cases} \frac{1}{(k-1)!} & \text{if } \{v_{i_1}, v_{i_2}, \dots, v_{i_k}\} \in E(G); \\ 0 & \text{otherwise.} \end{cases}$$

Let $\mathcal{D} = \mathcal{D}(G)$ be a kth order n-dimensional diagonal tensor, where $d_{i \dots i} = d_{v_i}$ for all $i \in [n] := \{1, 2, \dots, n\}$. Then $\mathcal{L} = \mathcal{L}(G) = \mathcal{D}(G) - \mathcal{A}(G)$ is the *Laplacian tensor* of the hypergraph G, and $\mathcal{Q} = \mathcal{Q}(G) = \mathcal{D}(G) + \mathcal{A}(G)$ is the *signless Laplacian tensor* of G.

Qi [15] showed that $\rho(\mathcal{L}(G)) \le \rho(\mathcal{Q}(G))$, and posed a question of identifying the conditions under which the equality holds. Hu et al. [9] proved that if G is connected, then the equality holds if and only if k is even and G is odd-bipartite. Here an even uniform hypergraph G is called *odd-bipartite* if $V(G)$ has a bipartition $V(G) = V_1 \cup V_2$ such that each edge has an odd number of vertices in both V_1 and V_2. Such partition will be called an *odd-bipartition* of G. Shao et al. [17] proved a stronger result that the Laplacian H-spectrum (respectively, Laplacian spectrum) and signless Laplacian H-spectrum (respectively, signless Laplacian spectrum) of a connected k-uniform hypergraph G are equal if and only if k is even and G is odd-bipartite. They also proved that the adjacency H-spectrum of G (respectively, adjacency spectrum) is symmetric with respect to the origin if and only if k is even and G is odd-bipartite. So, the non-odd-bipartite even uniform hypergraphs are more interesting on distinguishing the Laplacian spectrum and signless Laplacian spectrum and studying the non-symmetric adjacency spectrum.

Hu, Qi and Shao [10] introduced the *cored hypergraphs* and the *power hypergraphs*, where the cored hypergraph is one such that each edge contains at least one vertex of degree 1, and the kth power of a simple graph G, denoted by G^k, is obtained by replacing each edge (a 2-set) with a k-set by adding $k-2$ new vertices. These two kinds of hypergraphs are both odd-bipartite.