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Perfect state transfer on distance-regular graphs and association schemes

G. Coutinho [a,*], C. Godsil [a], K. Guo [b], F. Vanhove [c]

[a] Department of Combinatorics and Optimization, University of Waterloo, Canada
[b] Department of Mathematics, Simon Fraser University, Canada
[c] Department of Mathematics, Ghent University, Belgium

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ABSTRACT

We consider the representation of a continuous-time quantum walk in a graph X by the matrix $\exp(-itA(X))$. We provide necessary and sufficient criteria for distance-regular graphs and, more generally, for graphs in association schemes to have perfect state transfer. Using these conditions, we provide several new examples of perfect state transfer in simple graphs.

* Corresponding author.
E-mail addresses: gcoutinho@uwaterloo.ca (G. Coutinho), cgodsil@uwaterloo.ca (C. Godsil), krystalg@sfu.ca (K. Guo).

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1. Introduction

For a given graph X with adjacency matrix A, the transition matrix of the quantum walk at time t is given by $U = e^{itA}$. If u and v are distinct vertices of graph X, we say that X admits *perfect state transfer* if there is a time τ such that

$$U(\tau)e_u = \lambda e_v$$

for some $\lambda \in \mathbb{C}$.

The main problem we are concerned about is that of determining which graphs admit perfect state transfer. This problem is solved for paths and hypercubes (see [18]) and for circulant graphs (see [36]). It was also extensively discussed for cubelike graphs in general (see [10] and [15]). The effect of certain graph operations in perfect state transfer has also been considered in [2] and in [3]. A generalization to signed graphs is considered in [13]. Some recent surveys are [32] and [25].

In general, perfect state transfer is understood to be a relatively rare phenomenon. One of the first infinite families of graphs admitting perfect state transfer was given in [24]. In the known examples of graphs admitting perfect state transfer, the times at which perfect state transfer occur tend to be at $\frac{\pi}{2}$ or $\frac{\pi}{4}$. The known classes of graphs where perfect state transfer occurs at a time earlier than $\frac{\pi}{4}$ are the complete bipartite graphs $K_{2,n}$, examples from [14] and various examples given in this paper, including the Hadamard graphs.

In this paper, we will examine perfect state transfer in distance-regular graphs and, more generally, in graphs contained in association schemes. This problem was already studied in [30] and in [26], but here we will present a necessary and sufficient condition that can be easily tested. We determine precisely which graphs in the known infinite families of distance regular graphs admit perfect state transfer. In addition, for all graphs listed in [11, Chapter 14], we determine which admit perfect state transfer.

2. Quantum walks

Quantum walk is an important concept in quantum algorithms. A quantum walk is a quantum analog of a classical random walk. Quantum walk algorithms have been studied and shown to perform exponentially or polynomially better for various black box problems. We refer to [16,1,21,17] for further background on such results.

There are several ways of defining a quantum analog to a random walk. We focus on the *continuous-time quantum walk*, generated by the adjacency matrix of a graph. Let X be a graph with adjacency matrix A. The continuous-time quantum walk at time t is given by the unitary operator e^{-itA} and takes place in the Hilbert space whose elementary basis corresponds to the vertices of X. In particular, we address the problem of when X admits a transfer of state between two vertices without a loss of information with respect to a continuous-time quantum walk. We call this phenomenon *perfect state transfer* and we formalize it as follows.