Linear Algebra and its Applications 478 (2015) 274–293

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

An integral formula for multiple summing norms of operators ☆

Daniel Carando [a,b], Verónica Dimant [c,d], Santiago Muro [a,d,*], Damián Pinasco [e,d]

[a] Departamento de Matemática - Pab I, Facultad de Cs. Exactas y Naturales, Universidad de Buenos Aires, (1428) Buenos Aires, Argentina
[b] IMAS–CONICET, Argentina
[c] Departamento de Matemática, Universidad de San Andrés, Vito Dumas 284, (B1644BID) Victoria, Buenos Aires, Argentina
[d] CONICET, Argentina
[e] Departamento de Matemáticas y Estadística, Universidad Torcuato di Tella, Av. F. Alcorta 7350, (1428), Ciudad Autónoma de Buenos Aires, Argentina

ARTICLE INFO

Article history:
Received 19 September 2014
Accepted 24 March 2015
Available online 20 April 2015
Submitted by P. Semrl

MSC:
15A69
15A60
47B10
47H60
46G25

Keywords:
Absolutely summing operators
Multilinear operators

ABSTRACT

We prove that the multiple summing norm of multilinear operators defined on some n-dimensional real or complex vector spaces with the p-norm may be written as an integral with respect to stable measures. As an application we show inclusion and coincidence results for multiple summing mappings. We also present some contraction properties and compute or estimate the limit orders of this class of operators.

☆ This work was partially supported by CONICET PIP 0624, ANPCyT PICT 11-1456, ANPCyT PICT 11-0738, UBACyT 1-746 and UBACyT 20020130300052BA.

* Corresponding author.
E-mail addresses: dcarando@dm.uba.ar (D. Carando), vero@udesa.edu.ar (V. Dimant), smuro@dm.uba.ar (S. Muro), dpinasco@utdt.edu (D. Pinasco).

http://dx.doi.org/10.1016/j.laa.2015.03.031

Multiple summing operators
Stable measures

0. Introduction

The rotation invariance of the Gaussian measure on $\mathbb{K}^N$, which we will denote by μ_2^N, allows us to show the Gauss–Khintchine equality. It asserts that if $c_{2,q}$ denotes the q-th moment of the one dimensional Gaussian measure, and ℓ_2^N denotes $\mathbb{K}^N$ with the Euclidean norm, then for any $\alpha \in \mathbb{K}^N$, $1 \leq q < \infty$,

$$c_{2,q}\|\alpha\|_{\ell_2^N} = \Big(\int_{\mathbb{K}^N} |\langle \alpha, z\rangle|^q d\mu_2^N(z)\Big)^{1/q}. \tag{1}$$

We may interpret this formula as follows: the norm of a linear functional α on ℓ_2^N is a multiple of the L^q-norm of the linear functional with respect to the Gaussian measure on ℓ_2^N. One may ask if there is a formula like (1) for linear functionals on some other space, or even for linear or multilinear operators. For linear functionals, an answer is provided by the s-stable Lévy measure (see for example [6, 24.4]): for $s < 2$ there exists a measure on $\mathbb{K}^N$, called the *s-stable Lévy measure* and denoted by μ_s^N, which satisfies that for any $0 < q < s$, $\alpha \in \mathbb{K}^N$,

$$c_{s,q}\|\alpha\|_{\ell_s^N} = \Big(\int_{\mathbb{K}^N} |\langle \alpha, z\rangle|^q d\mu_s^N(z)\Big)^{1/q}, \tag{2}$$

where

$$c_{s,q} = \Big(\int_{\mathbb{K}} |z|^q d\mu_s^1(z)\Big)^{1/q}.$$

The question for linear operators is more subtle because there are many norms which are natural to consider on $\mathcal{L}(\ell_2^N)$. The first result in this direction is due to Gordon [9] (see also [6, 11.10]), who showed that the formula holds for the identity operator on ℓ_2^N, considering the absolutely p-summing norm of $id_{\ell_2^N}$, that is

$$\pi_p(id_{\ell_2^N}) = c_{2,q}\Big(\int_{\mathbb{K}^N} \|z\|_{\ell_2^N}^q \, d\mu_2^N(z)\Big)^{1/q}.$$

Pietsch [16] extended this formula for arbitrary linear operators from $\ell_{s'}^N \to \ell_s^N$, $s \geq 2$ and used it to compute some limit orders (see also [17, 22.4.11]).

To generalize the formula to the multilinear setting there is again a new issue, as there are many natural candidates of classes of multilinear operators that extend the ideal of absolutely p-summing linear operators (for instance the articles [12,14] are devoted to their comparison). Among those candidates, the ideal of multiple summing multilinear operators is considered by many authors the most important of these extensions