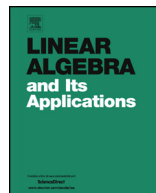




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Zero Jordan product determined algebras



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ABSTRACT

We prove that a unital algebra \mathcal{A} over a field of characteristic not 2 is zero Jordan product determined if it is generated by idempotents. Since an example of such an algebra is the matrix algebra $M_n(\mathcal{B})$ where $n \geq 2$ and \mathcal{B} is any unital algebra, this yields answers to questions posed in [4, p. 1492] and [7, p. 117].

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1. Introduction

Throughout this paper, let \mathcal{A} be a unital associative algebra over \mathbb{F} and \mathcal{X} be a linear space over \mathbb{F} , where \mathbb{F} is a field of characteristic not 2. In an algebra \mathcal{A} , we can define the Jordan product by $a \circ b = ab + ba$ for each a, b in \mathcal{A} .

\mathcal{A} is said to be *zero product determined* if every bilinear mapping ϕ from $\mathcal{A} \times \mathcal{A}$ into any linear space \mathcal{X} satisfying

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$$\phi(a, b) = 0, \text{ whenever } ab = 0$$

can be written as $\phi(a, b) = T(ab)$, for some linear mapping T from \mathcal{A} into \mathcal{X} .

Similarly \mathcal{A} is said to be *zero Jordan product determined* if every bilinear mapping ϕ from $\mathcal{A} \times \mathcal{A}$ into any linear space \mathcal{X} satisfying

$$\phi(a, b) = 0, \text{ whenever } a \circ b = 0$$

can be written as $\phi(a, b) = T(a \circ b)$, for some linear mapping T from \mathcal{A} into \mathcal{X} .

We denote by $\mathfrak{L}(\mathcal{A})$ the linear span of all idempotents in \mathcal{A} , and by $\mathfrak{J}(\mathcal{A})$ the subalgebra of \mathcal{A} generated by all idempotents in \mathcal{A} .

In [1,3–7], several authors study bilinear mappings through their action on zero product or zero Jordan product.

In [3], Brešar shows that if $\mathcal{A} = \mathfrak{J}(\mathcal{A})$, then \mathcal{A} is zero product determined. In [5], Ghahramani proves that if $\mathcal{A} = \mathfrak{L}(\mathcal{A})$, then \mathcal{A} is zero Jordan product determined.

In [4], Brešar et al. show that the matrix algebra $M_n(\mathcal{B})$ of $n \times n$ matrices over a unital algebra \mathcal{B} with $\frac{1}{2}$ is zero Jordan product determined for every $n \geq 3$. They ask whether the result is true for $n = 2$.

In Section 2, we improve the results in [3,5] through studying bilinear mappings on an algebra \mathcal{A} and show that \mathcal{A} is zero Jordan product determined if $\mathcal{A} = \mathfrak{J}(\mathcal{A})$. As applications, we affirmatively answer two questions posed in [4, p. 1492] and [7, p. 117].

2. Main results

Theorem 2.1. *If ϕ is a bilinear mapping from $\mathcal{A} \times \mathcal{A}$ into \mathcal{X} such that*

$$a \circ b = 0 \Rightarrow \phi(a, b) = 0$$

for all a, b in \mathcal{A} , then

$$\phi(a, x) = \frac{1}{2}\phi(ax, 1) + \frac{1}{2}\phi(xa, 1)$$

for all a in \mathcal{A} and x in $\mathfrak{J}(\mathcal{A})$. Thus \mathcal{A} is zero Jordan product determined if $\mathcal{A} = \mathfrak{J}(\mathcal{A})$.

Proof. By the definition of $\mathfrak{J}(\mathcal{A})$, we know that every x in $\mathfrak{J}(\mathcal{A})$ can be written as a linear combination of some elements x_1, x_2, \dots, x_k in $\mathfrak{J}(\mathcal{A})$ such that $x_k = p_{i_1}p_{i_2} \cdots p_{i_k}$, where $p_{i_1}, p_{i_2}, \dots, p_{i_k}$ are idempotents in \mathcal{A} . Since ϕ is bilinear, to show the theorem, it is sufficient to prove that

$$\phi(a, p_1p_2 \cdots p_n) = \frac{1}{2}\phi(p_1p_2 \cdots p_na, 1) + \frac{1}{2}\phi(ap_1p_2 \cdots p_n, 1) \quad (2.1)$$

for all a and idempotents p_i in \mathcal{A} .

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