

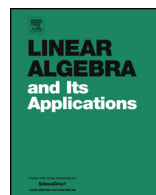


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Maps on matrices compressing the local spectrum in the spectrum



Hassane Benbouziane*, Mustapha Ech-Cherif El Kettani

Department of Mathematics, Faculty of Sciences DharMahraz Fes, University Sidi Mohammed BenAbdellah, 1796 Atlas Fes, Morocco

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ABSTRACT

Let $M_n(\mathbb{C})$ be the algebra of all complex $n \times n$ matrices. Let x_0 be a nonzero vector in \mathbb{C}^n . We describe maps ϕ on $M_n(\mathbb{C})$ (not linear or surjective) satisfying

$$\sigma_{T \pm S}(x_0) \subseteq \sigma(\phi(T) \pm \phi(S))$$

for all $T, S \in M_n(\mathbb{C})$. We also obtain a similar description by supposing that ϕ is surjective and $\sigma_{\phi(T) \pm \phi(S)}(x_0) \subseteq \sigma(T \pm S)$ for all $T, S \in M_n(\mathbb{C})$.

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1. Introduction and statement of the main results

Let $\mathcal{B}(X)$ be the algebra of all linear bounded operators on a complex Banach space X . The local resolvent set of an operator $T \in \mathcal{B}(X)$ at a point $x \in X$, denoted by $\rho_T(x)$, is the union of all open subsets $U \subseteq \mathbb{C}$ for which there exists an analytic function $f : U \rightarrow X$ such that $(T - \lambda)f(\lambda) = x$ for all $\lambda \in U$. The local spectrum of T at x is defined by

* Corresponding author.

E-mail address: benbhassan@yahoo.fr (H. Benbouziane).

$\sigma_T(x) := \mathbb{C} \setminus \rho_T(x)$ and is a (possibly empty) closed subset of $\sigma(T)$, the usual spectrum of $T \in \mathcal{B}(X)$.

The problem of characterizing spectrum-preserving maps on the algebra $M_n(\mathbb{C})$ of all complex $n \times n$ matrices was considered by a number of authors. In [19], Marcus and Moyls proved that if a linear map ϕ on $M_n(\mathbb{C})$ preserves eigenvalues (counting multiplicities), then there exists an invertible matrix $A \in M_n(\mathbb{C})$ such that

$$\phi(T) = ATA^{-1}, \quad (T \in M_n(\mathbb{C})), \tag{1}$$

or

$$\phi(T) = AT^tA^{-1}, \quad (T \in M_n(\mathbb{C})), \tag{2}$$

where T^t denotes, as usual, the transpose of $T \in M_n(\mathbb{C})$. This result has been generalized in different directions; see [4,12,14,20]. In particular, it is shown in [12,14] that for a map ϕ on $M_n(\mathbb{C})$ with $\phi(0) = 0$, the following statements are equivalent.

1. ϕ satisfies

$$\sigma(\phi(T) - \phi(S)) \subseteq \sigma(T - S), \quad (T, S \in M_n(\mathbb{C})), \tag{3}$$

2. ϕ satisfies

$$\sigma(T - S) \subseteq \sigma(\phi(T) - \phi(S)), \quad (T, S \in M_n(\mathbb{C})), \tag{4}$$

3. ϕ takes either the form (1) or (2).

This result has been shown before under additional assumptions such as continuity or surjectivity of the map ϕ . But as stated above, the equivalence between the last two statements is due to Costara [12] and the equivalence between the first and third statements was recently proved by Dolinar, Hou, Kuzma and Qi in [14].

The problem of describing linear or additive maps on $\mathcal{B}(X)$ preserving the local spectra has been initiated by Bourhim and Ransford in [8], and continued by several authors; see for instance [5–7,10,11,13,16,17]. In [4], Bendaoud, Douimi and Sarih used Costara’s approach and proved, for a fixed nonzero vector x_0 in \mathbb{C}^n , that a map ϕ on $M_n(\mathbb{C})$ with $\phi(0) = 0$ satisfies

$$\sigma_{\phi(T)-\phi(S)}(x_0) \subseteq \sigma_{T-S}(x_0), \quad (T, S \in M_n(\mathbb{C})) \tag{5}$$

if and only if there exists an invertible matrix $A \in M_n(\mathbb{C})$ such that $Ax_0 = x_0$ and $\phi(T) = ATA^{-1}$ for all $T \in M_n(\mathbb{C})$. They also show that the same conclusion remains valid when the reverse set inclusion in (5) occurs without surjectivity assumption on ϕ .

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