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On the distance Laplacian spectral radius of graphs

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We determine the unique graphs with minimum distance Laplacian spectral radius among connected graphs with fixed number of pendent vertices, the unique trees with minimum distance Laplacian spectral radius among trees with fixed bipartition, the unique graphs with minimum distance Laplacian spectral radius among graphs with fixed edge connectivity at most half of the number of vertices. We also discuss the minimum distance Laplacian spectral radius of graphs with fixed connectivity. For $k = 1, \dots, \lfloor \frac{n-2}{2} \rfloor$, we determine the unique n-vertex tree with the $(k+1)$-th smallest distance Laplacian spectral radius.

1. Introduction

We consider simple and undirected graphs. Let G be a connected graph on n vertices with vertex set $V(G)$ and edge set $E(G)$. For $u, v \in V(G)$, the distance between u and v in G, denoted by $d_G(u, v)$, is the length of a shortest path from u to v in G. The distance matrix of G is the $n \times n$ matrix $D(G) = (d_G(u, v))_{u,v \in V(G)}$. The spectrum of

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distance matrix, arisen from a data communication problem studied by Graham and Pollack [6] in 1971, has been studied extensively, see the recent survey [1]. For $u \in V(G)$, the transmission of u in G, denoted by $Tr_G(u)$, is defined to be the sum of distances from u to all other vertices of G, i.e., the row sum of $D(G)$ indexed by vertex u, i.e., $Tr_G(u) = \sum_{v \in V(G)} d_G(u, v)$. Let $Tr(G)$ be the diagonal matrix of vertex transmissions of G. The distance Laplacian matrix and distance signless Laplacian matrix of G are defined as $\mathcal{L}(G) = Tr(G) - D(G)$ and $\mathcal{Q}(G) = Tr(G) + D(G)$, respectively, see [2]. The spectrum of $\mathcal{Q}(G)$ has been studied to some extent, see [2,5,8–10]. Let $\lambda_1(G) \geq \cdots \geq \lambda_n(G)$ denote the spectrum of $\mathcal{L}(G)$, which we call the distance Laplacian spectrum of G [2]. Aouchiche and Hansen [2] showed that the distance Laplacian eigenvalues do not increase when an edge is added, and $\lambda_{n-1}(G) \geq n$ with equality if and only if the complement of G is disconnected. Aouchiche and Hansen [3] showed that for an n-vertex tree T with $n \geq 3$, $\lambda_1(T) \geq 2n - 1$ with equality if and only if T is the star, and computed the distance Laplacian characteristic polynomials of some graphs. Nath and Paul [7] characterized the n-vertex (connected) graphs G whose complements are trees or unicyclic graphs having $\lambda_{n-1}(G) = n + 1$, and showed that the largest distance Laplacian eigenvalue of a path is simple and the corresponding eigenvector has the similar property like that of a Fiedler vector.

The distance Laplacian spectral radius of G is the largest distance Laplacian eigenvalue of G, denoted by $\lambda(G)$. In this paper, we determine the unique graphs with minimum distance Laplacian spectral radius among connected graphs with fixed number of pendent vertices (vertices of degree one), the unique trees with minimum distance Laplacian spectral radius among trees with fixed bipartition, the unique graphs with minimum distance Laplacian spectral radius among graphs with fixed edge connectivity at most half of the number of vertices. We also discuss the minimum distance Laplacian spectral radius of graphs with fixed connectivity. For $k = 1, \ldots, \lfloor \frac{n-2}{2} \rfloor$, we determine the unique n-vertex tree with the $(k+1)$-th smallest distance Laplacian spectral radius.

2. Preliminaries

For vertex disjoint graphs G_1 and G_2, let $G_1 \cup G_2$ be the (vertex disjoint) union of G_1 and G_2, and $G_1 \vee G_2$ the join of G_1 and G_2, obtained from $G_1 \cup G_2$ by adding all possible edges between vertices in G_1 and vertices in G_2.

Let K_n be the complete graph on n vertices. Let nK_1 be the union of n vertex disjoint copies of K_1. Let $K_{a,b} = aK_1 \vee bK_1$ be the complete bipartite graph with a and b vertices in the two partite sets, respectively.

Let G be a graph. For $v \in V(G)$, let $N_G(v)$ or $N(v)$ be the set of neighbors of v in G. For two nonadjacent vertices u and v in G, let $G + uv$ be the graph obtained from G by adding the edge uv.

Lemma 2.1. *(See [2].) Let G be a connected graph with $u, v \in V(G)$. If u and v are nonadjacent in G, then $\lambda(G + uv) \leq \lambda(G)$.*